GEOGEBRA DYNAMIC SOFTWARE AS MATHEMATICAL MODELING SUPPORT IN ENGINEERING EDUCATION

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Abstract: Engineers have always represented a pillar of development of any society that strives towards further improvement and enhancement. Taking into account the importance of the role that engineers play in the development of modern society, it is essential that the education of future engineers be at the highest possible level. The perspective of engineering education tends towards functional knowledge as its main goal, which means preparing students to use their knowledge in a professional environment and to apply it to solving problems from various fields of engineering. In engineering, no matter its branch, mathematics has one of the most important roles because it represents the universal language of communication in science and engineering. Everyday engineering professional practice requires the application of many mathematical disciplines. Recent trends in engineering education aim at improving the mathematical skills of students, and significant efforts have been made toward creating new educational strategies for teaching mathematics in engineering. Computer technologies and mathematical modeling have a significant role in engineering education. Future engineers depend upon the experience of working with examples from real practice, which is often very hard to realize because of laboratory restrictions, expenses, and safety problems. In this paper, we propose a new approach to teaching mathematics in engineering, based on the use of mathematical modeling supported by computer technologies using the dynamical software GeoGebra. The aim of the authors of this paper was to explore students' perception of the influence of mathematical modeling supported by computer technologies, on creating positive attitudes towards mathematics and its usefulness in the engineering profession. For the purpose of the research mathematical model of motion problem was created in GeoGebra software. All key elements of the simulation of the motion problems' mathematical model were highlighted and discussed. The teachers' impressions after the teaching process clearly implied that students had a very positive attitude toward teaching using mathematical modeling and GeoGebra. This confirmed that students recognized the importance of the presented methods of approach and see the future of engineering education in the application of real examples and visualization in the classroom. Considering the directions of the further development of the presented method, the authors of this paper are strongly convinced that there are many fields of mathematics and engineering where the application of mathematical modeling and GeoGebra is possible and that there is a future for teaching approaches based on these principles.
Keywords: Engineering education, mathematical modeling, GeoGebra, motion problem.

1. INTRODUCTION

A society that seeks to improve and enhance has always relied on engineers to drive its development. Because engineers play a vital role in developing the modern world, it is imperative for future engineers to receive the best form of education. Engineering education emphasizes practical knowledge as its primary goal, meaning it aims to prepare students for careers in engineering and to apply their knowledge to solve various engineering challenges (Schuster, Richert & Jeschke, 2016).

Mathematics plays a crucial role in engineering, regardless of its branch, as it serves as a common medium of communication across both disciplines. Professional engineers are required to apply a variety of mathematical disciplines, including probability, algebra, and calculus, among others. Due to the importance of mathematical theory and its application in engineering, future engineers must receive a comprehensive mathematical education. Engineering education has been focusing on improving the mathematical proficiency of students and implementing new instructional strategies to teach mathematics to engineering students (Carr, Bowe & Ní Fhloinn, 2013).

Developing a clear understanding of the importance of mathematical knowledge, as well as its practical use in real-life situations, using actual examples from practice, as well as its importance in solving real-life engineering problems, is an essential component of the engineering curriculum. In addition, the constant advancement of technology requires the active integration of computer technology into every aspect of engineering education, as well as making teachers and students aware of all the potential applications of computer technology (Magana & de Jong, 2018; Sekulić, Borjanović & Popović, 2022).

Our paper proposes a new method for introducing mathematics to engineers, utilizing GeoGebra, a dynamical software program that supports mathematical modeling. It is our intention to use mathematical modeling as a method of illustrating mathematical concepts and motivating students, but also as a means of producing a mathematically productive outcome with genuine real-world motivation (Galbraith & Stillman, 2006). In this paper, the authors explore students' perceptions of computer-based mathematical modeling, focusing on the development of positive attitudes toward mathematics and its relevance to engineering professions. Our research involved the development of a mathematical model for motion problems, which was implemented in GeoGebra and demonstrated to engineering students. The goal of this paper is to demonstrate how GeoGebra software could be used in engineering education for mathematical modeling, and the students' perceptions of how effective such teaching methods are concerning mathematics, and how they will apply them in their studies and careers.

2. MATHEMATICAL MODELING AND GEOGEBRA IN ENGINEERING MATHEMATICS

Mathematics modeling is characterized by the fact that it uses mathematics to explain how real world phenomena interact. Because of these characteristics, engineers often use mathematical models to describe and analyze objects, as well as to experiment with and predict their behavior. Mathematical modeling is often taught and integrated into mathematics courses, before expanding into other disciplines. Using mathematical modeling to teach mathematics has been found to have positive effects on the students' understanding of mathematics and their ability to solve real-life problems (Schukajlow, Kaiser & Stillman, 2018).

Modeling involves several phases in which real world conditions are translated into mathematical representations. Each modeling cycle has a different number of phases, depending on its author. The most commonly used is the Galbraith-Stillman modeling cycle (Galbraith & Stillman, 2006), which consists of seven phases. A modeling cycle involves the following phases: first, gathering real-world data, transforming it into real-world problems, and building mathematical models based on those problems. In the next phase, mathematical solutions are created from the constructed mathematical model and interpreted in the real world. The mathematical model must match the given real-world problem for the solution to be considered valid and the report to be accepted and presented. If not, a revised solution is developed and the cycle of mathematical modeling from phase two is repeated.

Multimedia and computer-based teaching and learning have long been well-known for their benefits. The use of these teaching methods has been observed to enhance student’s knowledge and skills across all academic levels and disciplines. (Mayer & Moreno, 2002). As mathematical modeling is complex, it is generally believed that computer technologies should be used to implement mathematical modeling. Combining mathematical modeling with computer technology is thought to produce the best results because computers add a visual dimension to the process.

For the purposes of this paper, we used GeoGebra software. A dynamic mathematical software program, GeoGebra, is capable of applying numerous mathematical theories (Kostić & Sekulić, 2022; Sekulic *et al.*, 2020; Ziatdinov & Valles Jr., 2022). The main advantages of GeoGebra for educational purposes are its simplicity and low training requirements for teachers. Nonetheless, it is extremely powerful software, capable of responding to any request from users, especially during mathematical modeling. One of the reasons GeoGebra is the most commonly used modeling software is its dynamic nature. GeoGebra describes real-life problems through animations and simulations, while simultaneously building the mathematical background of the problem (Kostić, Sekulić & Spasić, 2020).

3. MATHEMATICAL MODEL OF THE MOTION PROBLEM IN A GEOGEBRA ENVIRONMENT

In research design, the authors of this paper have decided to, as an example, use motion problem because of its interdisciplinary nature. The mathematical model of motion problem can be used for teaching in different fields of mathematics, especially in differential calculus and definite integrals, but also in teaching physics and mechanics, more precisely, kinematics, which highly extends the scope of application of this mathematical model in engineering education.

For the purpose of research, a mathematical model of the motion problem was realized in *GeoGebra*. The reason for choosing *GeoGebra* software for realization of the mathematical model used in this study is that *GeoGebra* can support and display all the phases of mathematical modeling, starting from the real situation by using its dynamic effects, through mathematical description of the problem and its algebraic representation, to the final solution and mathematical model with which it can be further manipulated. The other, equally important reason for choosing *GeoGebra* as support software for mathematical modeling is its potential for showing dynamic and algebraic structure of the model at the same time.

In order to allow other teachers and students to use this mathematical model of motion problem for their educational, teaching and learning purposes, the authors of this paper have made it available on the official *GeoGebra* website (https://www.geogebra.org/m/AyrUWBM7).

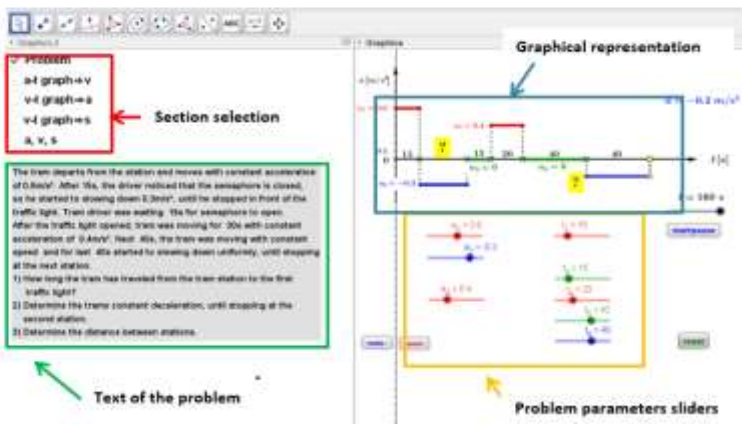
The presented mathematical model of the motion problem consists of five sections, each separately constructed for illustration of a specific part of the presented problem. Each section is available from other sections by checking its box in the upper left corner of the *GeoGebra* window (Figure 1).

Section I, marked with Problem, consists of two parts, called Graphics and Graphics 2 in GeoGebra. Graphic 2, on the left part of the screen, gives a detailed textual description of the motion problem, which is going to be modeled. The right part of the screen is a graphical representation of the considered problem.

The bottom part of the right side of the window, includes so-called sliders, special GeoGebra tools, which allow users to set an interval for values and the steps in this interval by which the value of a variable is taken. In that way, GeoGebra gives users the option to change parameter values, and by that to make simulations and animations. This characteristic makes GeoGebra the ideal software for the realization of the mathematical modeling process. In the movement problem, sliders are used for the interval representation of problem parameters (velocity, acceleration, deceleration, time, distance,...). Also, the user has the opportunity to change input parameters, by moving the sliders, and, in that way, create different movement problems and situations.

By clicking on the Start/Pause button, students can initiate the dynamic representation of the problem and in that way step by step visually follow the change of problem variables. In this way, students are given an insight into the visual/dynamic structure of the problem at the very beginning of solving process, thereby motivating them to consider all its aspects. On the other hand, this kind of representation shows the problem as a real situation and enables students to connect its mathematical and real-life representation and to apply principles of mathematical modeling in their learning experience. The leading idea for this kind of representation is to connect data and their mathematical representation with visual simulation.

Figure 1. Section I – Textual and graphical representation of the motion problem.



The Section II appears when checking the box *a-t graph → v* in the upper left corner of any window of *GeoGebra* simulation. Section II consists of two graphics, and its aim is to connect acceleration with velocity by calculating velocity using parameters of acceleration and time (Figure 2).

The left part of the window shows tables with data for time, acceleration, and velocity when calculated. The pink table shows time and velocity values, and the method of calculating velocity. The yellow table beneath shows the calculated value of the velocity at different moments of time. The bottom part on the left has two green-marked tables where step-by-step solutions with formulas for the time parameter (the left table) and for the acceleration parameter (the right table) are shown. In this part of *GeoGebra*, we used the standard formula $v=a\cdot t$ (v-velocity, a-acceleration, t-time) for the calculation of velocity.

The right side of the window gives a graphical representation of acceleration and velocity, and also by clicking the Start/Pause button it is possible to start the simulation of the whole process of calculating velocity when acceleration and time are changing following the initial values of the problem.

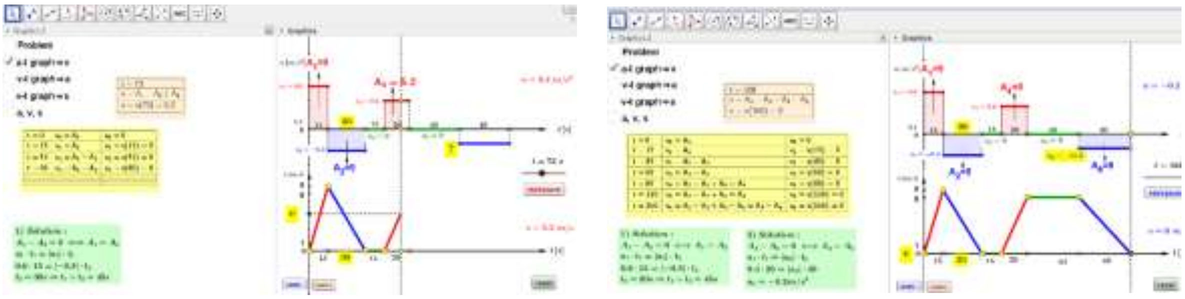
It is very interesting to mention the method of interpretation of the calculating process because this method was used intentionally to illustrate the connection between acceleration, time, and velocity.

The acceleration graph has acceleration on its ordinate and time on its abscissa, upper right corner of Section II (Figure 2). By calculating velocity, we actually calculate the area of rectangles appearing on the acceleration graph. These rectangles have a time parameter representing one side, and an acceleration parameter representing the other side. The lower graph represents velocity and its direct connection with acceleration. This method of velocity calculation, using rectangle areas, gives students one more opportunity to visualization of the presented problem and its parameters. At the same time, they can see one of the various ways to calculate velocity and later (in the framework of this *GeoGebra* representation) compare it with other methods from different fields of mathematics.

Figure 2 shows Section II in different moments of time, so we can demonstrate the dynamic nature of this mathematical model.

Figure 2. Section II – Calculating velocity out of acceleration and time – Different moments of time.

Time t=73 s *Time t=160 s*

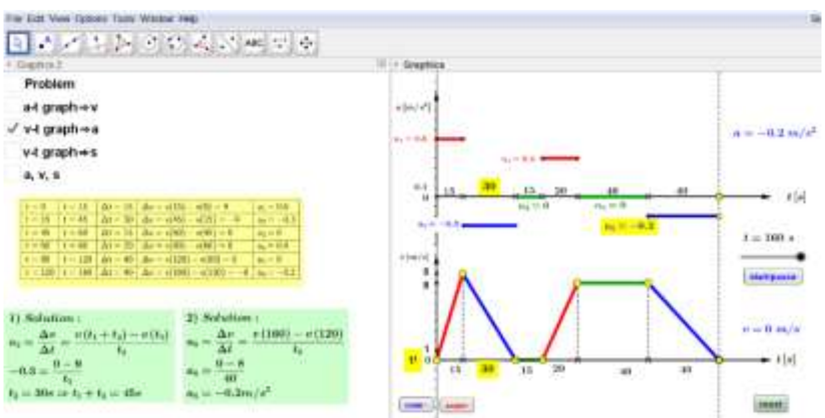


All sections of the motion problems' mathematical model realized in *GeoGebra* software have dynamic nature and work as simulations and animations. Consequently, this model is always active and students can observe the course of movement problems' solving process and make conclusions about its characteristics and connections with other fields of engineering.

Section III illustrates acceleration, its calculation, and its connection with velocity and time. When the user checks the box marked with *v-t graph* $\rightarrow$ *a*, it automatically follows to the Section III window. This section window is also divided into two parts. The one on the left side presents data parameters in a table, and on the right side, the change of parameters is shown as a simulation (Figure 3).

The left side of the window in Section III has two integrated tables. The yellow one shows, as in the previous section, the calculated values of acceleration in different moments of time. Two green marked tables on the bottom of the left side of the window provide the overview of step by step solution with formulas for the time parameter (the left table) and for the acceleration parameter (the right table), but this time using derivative as the method of solving the problem (we calculated velocity as the change of distance over the change of time and the acceleration as the change of velocity over the change of time).

Figure 3. Section III – Calculating acceleration out of velocity and time.

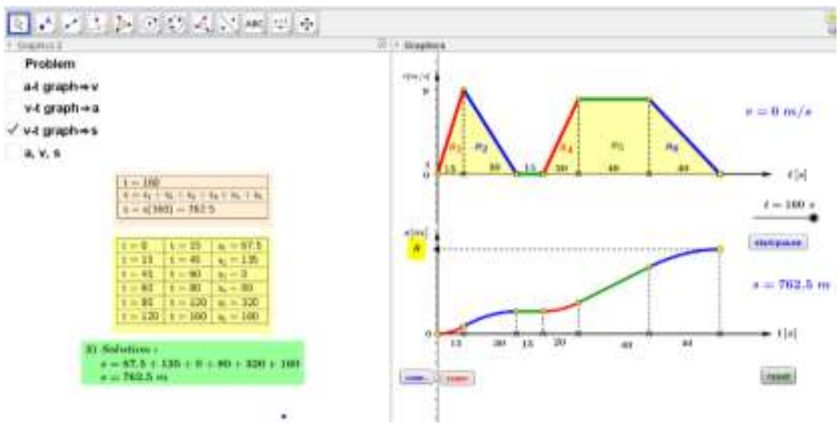


The right side of the window gives a graphical representation of acceleration (upper graph) and velocity (bottom graph) over time. This representation is especially suitable for teaching derivatives and their properties because by comparing two presented graphs it can be observed how the original function is connected with its derivative function.

Section III is constructed in order to involve different mathematical theories in this model because we wanted to demonstrate the application of differential calculus in engineering and to show its dynamic and visual nature by using the *GeoGebra* model. Calculating acceleration in this way, we added one more dimension to the mathematical model of the movement problem derivative. This mathematical model can be used for teaching not only movement problems, but also as an illustration of derivative properties and their application to real-world problems, which is essential for future engineering professionals.

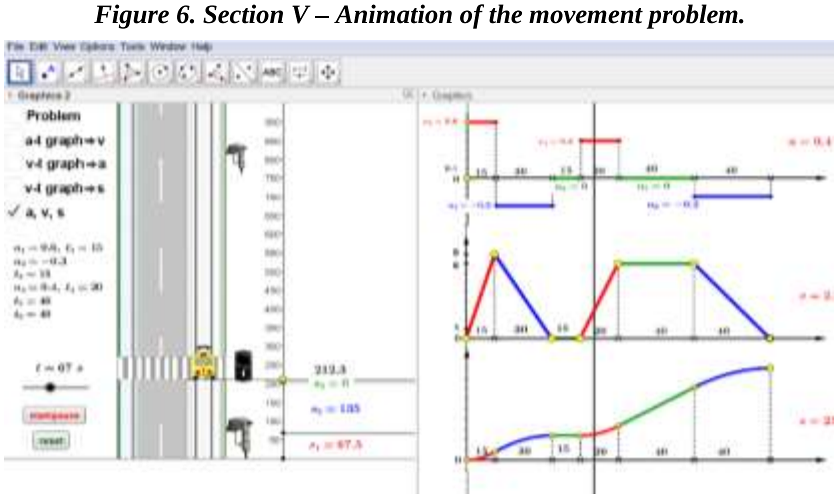
Figure 4 illustrates Section IV where the solution to part of the movement problem concerning the covered distance is shown. Using velocity and time as parameters, the distance was calculated. The right side of the Section IV window is a graphical representation of distance and its calculation mode. The upper graph on the right side of Section IV shows velocity as a function of time (Figure 4). In order to calculate distance, we calculated it as the area of triangles (rectangles) determined by the velocity graph. This time we calculated distance as a definite integral of velocity with respect to time. In that way integrals are also part of the mathematical model used in this research, which broadens the scope of its application. The bottom graph shows the distance as a function of time and its connection with velocity on the upper graph can be observed in this part of the *GeoGebra* motion problem model.
The left side of the Section IV is reserved for the tabular display of the parameters. The pink table shows time and distance values, the yellow table shows the calculated distance in different moments of time and the green table contains step-by-step solutions for distance.

Figure 4. Section IV – Calculating distance



This section introduced one new method in movement problem solving by using definite integrals. In this way, we have combined several mathematical theories (geometry in calculating areas, derivatives and definite integrals, function graph and its properties) with physics/kinematics (movement problem) with the intention to show to students all aspects of the interdisciplinary nature of one problem and its solution.
Section V unites all four previous sections, and through graphical simulation and animation illustrates the presented problem. Figure 6 gives Section V window overview, with an emphasis on the left side of the window, where an animation of the presented problem can be seen which fully corresponds to the textual representation of the problem from Section I. This animation represents a central part of the mathematical model of the motion problem and also of the aim of this research - to apply and introduce new methods in mathematics education, using mathematical modeling and computer technologies.

Figure 6. Section V – Animation of the movement problem.



Right side of the window is a graphical simulation of acceleration, velocity, and distance and by their simultaneous change students can observe the dependency of these three concepts. Also, from graphical simulation, it can be seen that different methods and theories can be used for the calculation of acceleration, velocity, and distance and that all of them interact with one another. For the purpose of the realization of motion problems' simulation, we used the area of triangle and rectangle, derivative and definite integral. Using different theories for describing mathematical models of motion problems, the authors of this paper wanted to achieve the possibility of implementing this mathematical model on different levels of mathematics and especially engineering education.

The mathematical model of motion problem, with all its simulations, calculations, and animations was implemented in the teaching process and presented to students in order to obtain their opinions and impressions about its usefulness and produced effects.

4. RESULTS AND DISCUSSION

The mathematical models and the GeoGebra materials presented in this paper were designed and constructed according to the requirements of the modern methodology of mathematics education. They were applied in the teaching process in order to help both, the students and the teachers.

The work with the presented GeoGebra material has showed that the teaching process was more than successful, according to students' reactions and teachers' impressions. The students' reactions were positive to the presented materials, they were content with the opportunity to learn using real-life problems, and their main conclusion on this type of learning process was that they liked the most that they could observe and manipulate the objects in the materials, creating different situations which they could explore.

On the other hand, the teachers' impressions were also in favor of using mathematical modeling and GeoGebra dynamical materials. The teachers' comments about this kind of teaching process were that the students were more involved during the class and that they interact in a better way with each other and with the teacher than before when they were taught using traditional teaching methods.

Considering all mentioned, it could be confirmed that the teaching process using the mathematical model of the motion problem realized in the GeoGebra environment was successful and that both, teachers and students agree that this kind of teaching and learning can benefit all of them.

5. CONCLUSIONS

Mathematical modeling and computer technology play an important part in engineering education. In order to be successful in the engineering field, future engineers need to be able to apply experiences from real-life practice, which can sometimes be difficult due to laboratory restrictions, costs, and safety concerns.

Real-time representations and simulations of real processes can be achieved through the combination of computer technology and mathematical modeling. Furthermore, it can assist students in gaining a deeper understanding of the nature of the real processes as well as provide an opportunity for them to experiment with many engineering problems outside of the limitations listed above.

In the future, we plan to further develop teaching materials, in order to improve them by adding more simulations and animations of real processes, so the students could gain skills for observing, solving problems, and applying the mathematical tools in real engineering practice.

REFERENCES

Carr, M., Bowe, B., & Ní Fhloinn, E. (2013). Core skills assessment to improve mathematical competency. *European Journal of Engineering Education*, 38(6), 608-619.

Galbraith, P., & Stillman, G. (2006). A framework for identifying student blockages during transitions in the modelling process. *ZDM*, 38(2), 143-162.

Kostić, V., & Sekulić, T. (2022). Teaching Quadratic Functions in Classroom and Online Using Mathematical Software Tools. Proceedings of the 9th International scientific conference Technics and Informatics in Education – TIE 2022, University of Kragujevac, Faculty of Technical Sciences Čačak, 315-319.

Kostić, V., Sekulić, T., & Spasić, A. (2020). Mathematical Modeling and GeoGebra in Engineering Education, Proceedings of Academy of Applied Technical and Preschool Studies Nis, Serbia, 105-108.

Magana, A. J., & de Jong, T. (2018). Modeling and simulation practices in engineering education. *Computer Applications in Engineering Education*, 26(4), 731-738.

Mayer, R. E., & Moreno, R. (2002). Aids to computer-based multimedia learning. *Learning and instruction*, 12(1), 107-119.

Sekulić, T., Borjanović, I., & Popović, D. (2022). Applied Mathematical Software Tools for Engineers at Technical College of Applied Sciences in Zrenjanin. Proceedings of the 8th International Conference on Industrial

Engineering SIE 2022, University of Belgrade, Faculty of Mechanical Engineering, Industrial Engineering Department, Beograd, 1-4.

Sekulic, T., Takaci, Dj., Strboja, M., & Kostic, V. (2020). Influence of Mathematical Modeling in GeoGebra Environment on Learning Derivative. *The International Journal for Technology in Mathematics Education (IJTME)*, Vol. 27, No 2, 61-82.

Schuster, K., Richert, A., & Jeschke, S. (2016). New Perspectives for Engineering Education–About the Potential of Mixed Reality for Learning and Teaching Processes. In *Automation, Communication and Cybernetics in Science and Engineering* 2015/2016 (pp. 407-416). Springer, Cham.

Schukajlow, S., Kaiser, G., & Stillman, G. (2018). Empirical research on teaching and learning of mathematical modelling: a survey on the current state-of-the-art. *ZDM*, 50(1-2), 5–18.

Ziatdinov, R., & Valles Jr, J. R. (2022). Synthesis of modeling, Visualization, and programming in GeoGebra as an effective approach for teaching and learning STEM topics. *Mathematics*, 10(3), 398.