THE INFLUENCE OF THE VIBRATO EXTEND ON THE INHARMONICITY FACTOR TO THE STRINGED MUSICAL INSTRUMENTS

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Abstract: This paper analyzes the effect of the vibrato technique on inharmonicity of the tone played on stringed musical instruments. In the first part of the paper, the inharmonicity of the stringed musical instruments, which is a consequence of the irregular oscillation of the tensioned string, is described. First, the inharmonicity factor *β*, which is a measure of inharmonicity, is defined. After that, the parameters for tones with vibrato effect: intonation (f_i), Vibrato Extend (VE) and Vibrato Rate (VR), are defined. In the second part of the paper, the Experiment is described. In the Experiment, the effect of Vibrato Extend VE on the inharmonicity factor *β* was analyzed. Tones were played on the Fender Stratocaster electric guitar using the vibrato technique, with the B4 starting tone. The all played tones were recorded in the form of a *wav* file (musical signals), and, from them, the Test Base was formed. The results of the Experiment are shown graphically. The change of the tension force of the string, *F*, as well as the dimensions of the string (length *L*, diameter *d*), which occurred as a result of the transferal displacement *h* of the string, i.e. Vibrato Extend, was analyzed. By applying the Fourier transform, the amplitude characteristic of musical signals is determined. After that, the positions of the fundamental frequency f_0 and its harmonics f_k, were analyzed. Based on the analysis, the inharmonicity factors *β* for all displacements *h*, i.e. for Vibrato Extend, were calculated. Through statistical analysis, the analytical function, that connects the inharmonicity factor *β* with Vibrato Extend, was calculated. Finally, it was shown that increasing the Vibrato Extend leads to a decrease in the inharmonicity factor.
Keywords: Fundamental frequency. Harmonic. Inharmonic, Vibrato Extend. Vibrato Rate.

1. INTRODUCTION

The stringed musical instruments produce tone, that is, generate an acoustic wave, by oscillating the string (Kakarwal & Chaudhary, 2020). The string oscillates with the fundamental frequency f_0. The fundamental frequency depends on: a) the dimensions of the string (length *L*, diameter *d*), b) the material of the string and c) the tension force *F*. In addition to the fundamental frequency f_0, as a consequence of the complex oscillation of the string (the appearance of waves at 1/2, 1/4, 1/8, ... the length of the string *L*), the acoustic components of the waves, at frequencies that are integer multiples of the fundamental frequency ($f_k = k \cdot f_0$, k = 1, 2, ...), are generated (Bhalke et al., 2014). Spectral components that are integer multiples of the fundamental frequency are called *harmonics* (Elvander & Jakobsson, 2020). Music theory describes the complex oscillation of the string and the generated tone using the term *aliquots* (lat. *aliquoties* - several times). Aliquots are the harmonic components of the generated tone. A detailed mathematical analysis shows that, in the composition of one tone, through aliquots, all tones are contained. This means that the aliquots of various tones overlap each other. The different number of aliquots, as well as their different strength in the total sound, determine the color of the sound, that is, the tone (Milivojević & Balanesković, 2009). Aliquots are also called partial tones, i.e. *partials* (Milivojević et al., 2016).
A more detailed analysis of the string oscillation shows that, due to the string parameters and tension force, the harmonic frequencies are not integer multiples of the fundamental frequency. This phenomenon is called inharmonicity (Elvander et al., 2017). The inharmonicity of the string oscillation is represented by the inharmonicity factor *β*. Inharmonicity necessarily leads to distortion of the aliquotity of the played tone (Lixin et al., 2009). The quality of stringed musical instruments can, in addition to other parameters, be shown using the degree of inharmonicity β. In the case of a piano, the strings are tightened with a large force *F*, which results in reduced elasticity of the string. The consequence is that the frequency positions of the partials f_k are in the positions of non-integer multiples of the fundamental frequency ($k \cdot f_0$). Therefore, a musical instrument with such strings is not

harmonic, but inharmonic (Milivojević et al., 2014). The increase in inharmonicity of the instrument, in addition to the elasticity of the string, is also influenced by the acoustic impedance of the piano's resonator plate. In the case of the guitar, it is the resonator body (Berenguer et al., 2006).

In order to increase the expressiveness of the played or sung tone, the vibrato technique is applied. The vibrato is a natural expression of the musician's emotions, understanding of music and musical philosophy (Wakazama et al., 2004). Many scientific disciplines, such as: psychology, musicology, signal processing, etc., study vibrato. On the acoustic side, the vibrato represents regular fluctuations of the fundamental frequency (Arroabarren & Rodet, 2006). In the stringed instruments, vibrato is created by changing the length of the string during tone reproduction. This changes the frequency of the reproduced sound, i.e. frequency modulation is performed (Liu et al., 2021). In addition to frequency modulation, amplitude modulation also occurs. All these effects affect the quality and experience of vibrato (Migata et al., 2010). Realization of the vibrato tone in the guitar is realized by pressing of the string on the corresponding fret, and after that, the transferal movement of the string. Therefore, there is an increase in the tension force, which causes a change in the length and diameter of the string. Changing the length and diameter of the string, as well as the tension force, causes a change in the inharmonicity of the played tone. Vibrato is defined using parameters: a) intonation (fundamental frequency f_0 of the tone), b) Vibrato Extend VE (frequency shift in relation to the fundamental frequency), and c) Vibrato Rate VR (rate of change of the fundamental frequency) (Milivojević & Balanesković, 2012).

In this paper, the authors try to answer the question *is there a relationship between the inharmonicity factor and the intensity of the vibrato effect, that is, from Vibrato Extend*? For this purpose, the Experiment was carried out. For the purposes of the Experiment, a Test Base, which is composed of recorded tones, was created. On the electric guitar (Fender Stratocaster) tone B4 (first string, seventh fret) with vibrato effect was realized. The vibrato effect was realized by transversely moving the string along the fret, with the displacement *h*. Transverse displacement *h* directly affects Vibrato Extend. Therefore, as a consequence of the change on the tension force *F*, the length *L* and the diameter of the string *d*, the inharmonicity factor of the string, has changed. The results of the Experiment (wire dimensions *L* and *d*, tension force *F*, fundamental frequency f_0, cent frequency c_0, inharmonicity factor β) are shown using graphs. By comparative analysis of experimental results and application of numerical methods (approximation, fitting), the formula, which connects the coefficient of inharmonicity with the transverse displacement of the string, i.e. VE, was determined. The formula is created using a polynomial of the third degree.

The further organization of the paper is as follows. Section 2 describes the inharmonicity of stringed musical instruments. In Section 3, the vibrato parameters are described. Section 4 presents the Experiment and the analysis of the experimental results. Section 5 is the Conclusion.

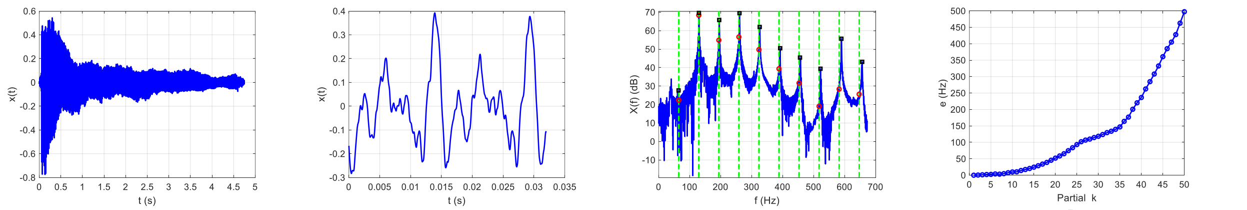
2. INHARMONICITY OF STRINGED MUSICAL INSTRUMENTS

The theory of music implies harmonicity in defining the frequency composition of a tone, i.e. that the harmonics (partials) are the integer multipliers of the fundamental frequency, which mathematically can be presented as $f_k = k \cdot f_0$, where f_0 is the fundamental frequency, k = 1, 2, ... is the ordinal number of a partial and f_k the frequency of the partial. The frequency shifting of the partial from the frequency position of the harmonic represents the inharmonicity of a tone. The frequency of the k-th partial is $f_k = k \cdot f_0 \cdot \sqrt{1+\beta \cdot k^2}$, where β is the coefficient of inharmonicity. The factor of inharmonicity β depends on the material the string is made of and can be calculated on the base of:

$$\beta = \frac{\pi^3 \cdot Q \cdot d^4}{64 \cdot L^2 \cdot F}, \tag{1}$$

where *Q* is Jung's modul of elasticity of the material the string was made of, *d* the diameter of the string, *L* the length of the string and *F* the tension force.

The inharmonicity of the oscillating string results in the generated tone of the string musical instrument (Milivojević & Brodić, 2013). The consequences of the inharmonicity are shown in Fig.1. As an example, the C2 tone of the Steinway B piano (fundamental frequency f_0 = 65.406 Hz) was analyzed. Figure 1.a shows the time form of the electrical signal, which is related to the C2 tone. For the purposes of further analysis, the signal is divided into blocks of 32 ms duration (Fig. 1.b). Fig. 1.c shows the spectrum of the C2 tone ($\beta = 1.28 \cdot 10^{-4}$). The vertical green lines and the red symbol 'o' show the spectral positions of the expected harmonics (partials), while the black symbols '□' show the real positions of the partials (inharmonics). It is observed that the positions of real harmonics and expected harmonics are different. In Fig. 1.d shows the harmonic error, $e(k) = f_{ph} - k \cdot f_0$, where f_{ph} is the frequency of the k-th harmonic component and k = 1, 2, ... 50, and which is a consequence of inharmonic oscillation of the string.



a) b) c) d)

Figure 1 – C2 tone (F0 = 65.406 Hz): a) whole signal, b) segment 32 ms, c) partial positions, and d) harmonicity error, e.

3. VIBRATO

Vibrato represents a periodic change in the fundamental frequency of the played musical tone to higher and lower levels in relation to the intonation pitch. In accordance with the previous definition, the following vibrato parameters are defined for the played vibrato note: a) intonation (pitch, i.e. fundamental frequency f_0), b) Vibrato Extent VE (depth of displacement of the fundamental frequency in direction of the intonation frequencies of the neighboring tones) and c) Vibrato Rate VR (rate of change of the fundamental frequency) (Milivojević & Balanesković). Using mathematical notation, a periodic signal with varying amplitude and frequency can be represented in the form $s(t) = A_i(t)\cdot\cos(\phi(t))$, where $A_i(t)$ is the instantaneous amplitude, and $\phi(t)$ is the instantaneous phase. Instantaneous frequency $f_i(t) = \partial(\phi(t)) / \partial t$, can be represented in the form $f_i(t) = b(t) + a_i(t)\cdot\cos(\phi_i(t))$, where a, b and ϕ are parameters. It is possible to establish a relationship between parameters of the instantaneous frequency and parameters of the vibrato tone : a) intonation $\leftrightarrow b$, b) $a_i \leftrightarrow$ VE, and c) $\phi \leftrightarrow$ VR.

Figure 2 shows the trajectory of the vibrato tone played on the violin. For the purposes of objective analysis of the vibrato parameters of the musical signal, a logarithmic scale, where the frequencies are displayed in cents, was used. Cents are determined based on the frequency scale in Hz: $c = 1200 \cdot \log_2(f / F_n)$, where c is the frequencies in cents, f is the instantaneous frequency (Hz) and F_n is the normalization frequency (Hz). In this paper, all examples are presented with tone A4 as a normalization tone (F_n = 440 Hz). The trajectory of the fundamental frequency of tone A4 is shown in fig. 2 and marked with (a). The intonation is presented as the mean value of the fundamental frequency trajectory (fig. 2. (b)). The mean value is determined for the entire time interval of the duration of the vibrato tone (fig. 2. (c)). According to the markings in fig. 3. the mean values of the parameters VR and VE are defined as follows: $VR = N \Big/ \sum_{n=1}^{N} R_n$ and $VE = \frac{1}{N}\sum_{n=1}^{N} E_n$ (Milivojević & Balanesković, 2012).

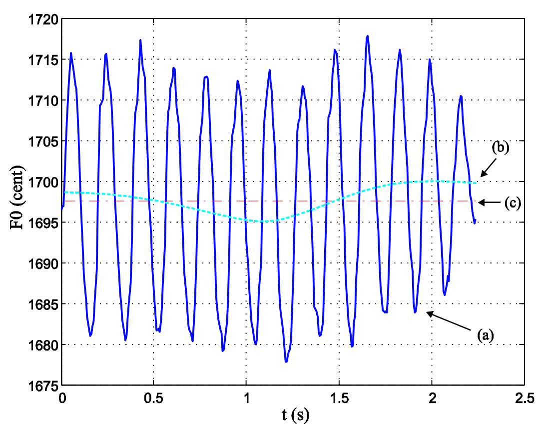


Figure 2 – Trajectories: a) fundamental frequencies, b) intonation and c) mean value of the trajectory of the fundamental frequency of the vibrato tone.

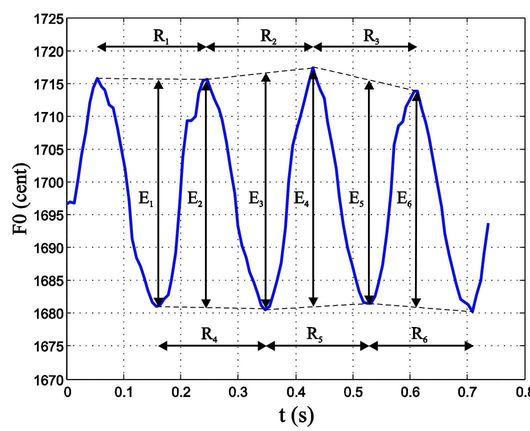


Figure 3 – Characteristic values of the trajectory of the fundamental frequency of the vibrato tone, which are used to determine the VR and VE parameters.

4. EXPERIMENTAL RESULTS AND ANALYSIS

Experiment

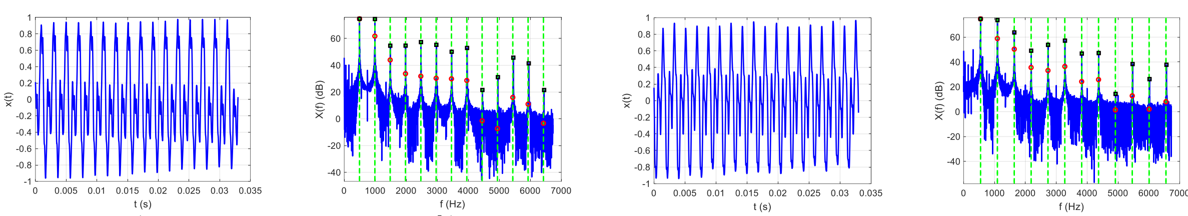
The factor of inharmonicity β in the electric guitar was determined experimentally. The aim of the Experiment is to determine the inharmonicity factor in the vibrato technique of playing tones. First, the tone A4 is played. The A4 tone is played by pressing the E4 string with finger on the seventh fret. After that, a transverse movement of the pressed string along the fret, for step of displacement Δh, was made, and then the tone was played. This process of moving the pressure point of the string along the fret and playing the tone has been repeated several times. All played tones are recorded. In this way, the Test Base was created. After that, the algorithms for estimating the fundamental frequency of the played tones form Base were applied. The inharmonicity factor β for each displacement h is calculated. In addition, the Vibrato Extent VE was calculated for each displacement h. The results of the Experiment are shown graphically.

Finally, the Experimental results were analyzed, and the analytical dependence of the inharmonicity factor β on: a) displacement h and b) Vibrato Extent VE was calculated. The Fender Stratocaster guitar was used in the Experiment. Step of displacement is Δh = 1mm. N = 21 tones were played and recorded. The maximum displacement is h_{max} = 20 mm. Length of the string L = 425 mm. The tension force of the wire is F = 72.2 N.

Test Base

The Test Base was formed by recording tones played on an electric Fender Stratocaster guitar. On the first string, on the seventh fret, a B4 tone is played. By transversely moving the string along the fret, for displacement h, tones are produced. A total of 21 tones were played. The strings are a product of the D'Addario company, which is a leader in the field of guitar string production, made of nickel, type EXL 110 (high temper, high carbon steel wire). String dimensions: length L = 425 mm (up to the seventh fret) and diameter d = 0.25 mm. Recording of played tones was done with F_s = 44100 Hz and 16 bps. In Fig. 4 shows the time forms and spectral characteristics of the musical signal for the two positions of the string pressure on the seventh fret, namely for: a) minimum displacement h = 0 mm (time form fig. 4.a, spectral characteristic fig. 4.b) and b) maximum displacement h = 20 mm (time form fig. 4.c, spectral characteristic fig. 4.d).

Figure 4 – Tone for displacement h = 0 mm : a) time form i b) spectral characteristics . Tone for displacement h = 20 mm : c) time form i d) spectral characteristics ..

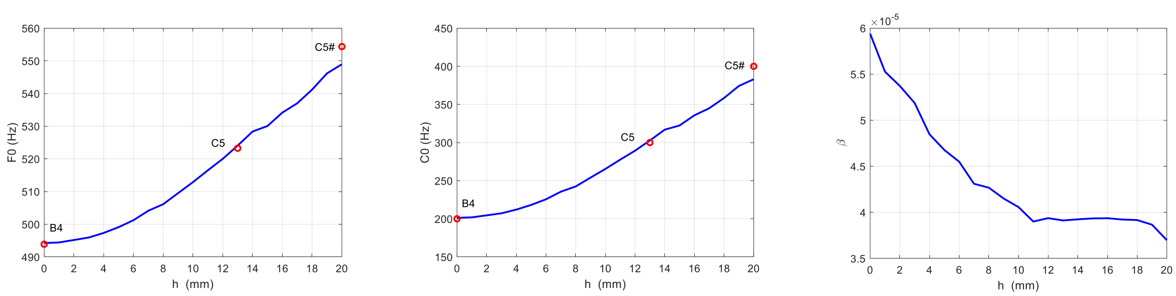


a) b) c) d)

Results

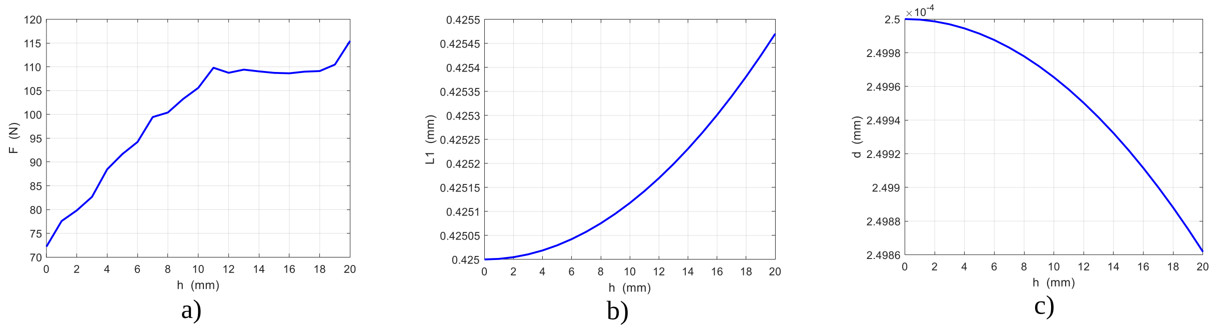
As a result of the transverse displacement of the pressure point on the string along the seventh fret ($h = 0 : \Delta h : 20$ mm), there was a change in: a) the tension force F (fig. 5.a), and b) the dimensions of the string (length L (fig. 5 .b) , diameter d (fig. 5.c)). The change in pitch, that is, the fundamental frequency f_0, is shown in fig. 6. a. In fig. 6.b shows the fundamental frequency c_0 in the cent scale. The inharmonicity factor β is shown in fig. 6. c.

Figure 5 – Changes due to transverse displacement h: a) tension force, b) length L and c) diameter d of the string.



a) b) c)

Figure 6 – Changes due to transverse displacement h: a) fundamental frequency fo, b) fundamental frequency c_0 and c) inharmonicity factor.



Analysis of results

Based on the results of the Experiment, which are shown in fig. 5, it is concluded that there was a change in: a) tension force *F* (fig. 5.a) and b) dimensions (length (fig. 5.b), diameter (fig. 5.c)) of the string. These changes necessarily cause a change in: a) the fundamental frequency (fig. 6.a and fig. 6. b) and b) the inharmonicity factor of the played tone fig. 6.c (Eq. (1)). The maximum change in fundamental frequency is represented as Vibrato Extend VE. The dependence of VE on displacement *h* is shown in fig. 7.a. It can be concluded that the dependence of VE on *h* is largely linear. Using the numerical method of polynomial fitting, an experimental, third order function, which describes the dependence of β on *h*, was determined:

$$\beta_{est}\left(h\right)=a_3\cdot h^3+a_2\cdot h^2+a_1\cdot h+a_0\,, \qquad (2)$$

where a_3 = -0.00004·10^{-4}, a_2 = 0.00205·10^{-4}, a_1 = -0.03513·10^{-4} and a_0 = 0.59546·10^{-4}. In fig. 7.b shows the inharmonicity factor β, obtained: a) experimentally (β_{exp}) and b) using the formula (eq. (2)) (β_{est}). Dependence of the inharmonicity factor β on VE, approximated by a third order polynomial function:

$$\beta_{est}\left(VE\right)=b_3\cdot VE^3+b_2\cdot VE^2+b_1\cdot VE+b_0\,, \qquad (3)$$

where b_3 = 0.0001·10^{-4}, b_2 = 0.0005·10^{-4}, b_1 = -0.0166·10^{-4} and b_0 = 0.5545·10^{-4}.

In Fig. 7.c shows the inharmonicity factors: a) experimental (β_{exp}) and b) determined using the formula (eq. (3)) (β_{est}).

It is concluded that, with an increase in the vibrato depth VE, which is directly related to the transfer displacement *h*, the inharmonicity of the tone decreases. A very interesting conclusion is that, with large displacements, that is, a large VE, a tone is played that overlaps the neighboring semitone (fig. 6.b, tone B4 tone C5). In extreme cases, where the technical possibilities of the instrument allow it, it is possible to overlap the vibrato tone with the neighboring tone (fig. 6.b, tone B4 tone C5#). The fact that, with the increase of Vibrato Extend VE, the inharmonicity of the tone decreases, as well as the pitch of the tone approaches the neighboring tones, can be explained by the fact that, in the listener's mind, the vibrato tone is perceived as pleasant.

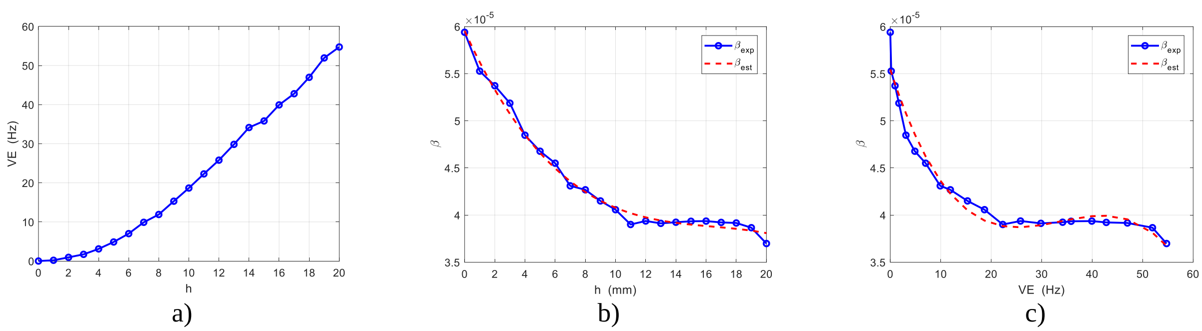


Figure 7 – a) dependence of VE on displacement h, b) experimentally inharmonicity factor β_{exp} and c) determined inharmonicity factor β_{est}.

5. CONCLUSION

In the paper, the inharmonicity of stringed instruments, which arises as a consequence of irregular string oscillation, is analyzed. The parameters of the vibrato (VE and VR), which is used as an ornament of the played note, are described. When playing a note with the vibrato technique, a change in the tension force F of the string, as well as a change in the length L and diameter d of the string, are the consequences. Due to this effect on the string, there is a change in the inharmonicity factor β of the string. In the Experiment, which is described in the second part of the paper, the effect of playing the note B4, with the vibrato technique on the string, was analyzed. In the Experiment, the note B4 was played on a Fender Stratocaster electric guitar. Using numerical approximation and fitting, a third-order polynomial function, which describes the dependence of the inharmonicity factor β from Vibrato Extend (VE) parameter, was determined. The analysis of the results, as well as the approximation function, indicates the fact that, with the increase of the VE parameter, the inharmonicity of the vibrato tone decreases. The combined of the vibrato effect, which represents the ornament of the note, and the reduction of inharmonicity, causes a pleasant feeling of the note.

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