

METHODOLOGY OF LEARNING OF ADDING AND SUBTRACTING NUMBERS UP TO 20 IN THE NEW BULGARIAN MATHEMATICS FOR THE FIRST GRADE

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Abstract: Over the last three years, educational reform has taken place in Bulgaria. It also affects the mathematics curriculum in the first grades. The curriculum for mathematics for the first grade was approved by Order of the Minister of Education and Science No RD 09-1857 dated 17.12.2015. It came into force in the school year 2016/2017. One of the areas of mathematical competence in the first grade curriculum is “Numbers”. It is basic. In this area of competence, students are expected to build up their knowledge of the numbers up to 10 that they have from the preparatory group/class. Students should understand concepts for numbers up to 10, up to 20 and numbers 30, 40, 50 ... 100. It is intended that they master adding and subtracting the numbers up to 10 to automatism. With numbers up to 20, pupils in the first grade have to master adding and subtracting without carrying over the ten to automatism. At the end of the school year, students study adding and subtracting with the numbers 10, 20, 30, 40, 50 ... 100. During the second semester of the first grade, they mainly study to add and subtract the numbers up to 20 by carrying over the ten. These are tasks of type $8 + 5$, $11 - 4$. These tasks are of great importance as they are the basis of the adding and subtraction with carrying over the ten with larger numbers – up to 100, up to 1000 and over 1000 (like $48 + 25$; $328 + 535$; $234,158 + 413,625$). If children do not master the adding and subtraction with numbers up to 20 with carrying over the ten to automatism, they have no chance to learn to add and subtract the larger numbers with carrying over. That is why, in textbooks, most time of the learning process and most topics of this learning content are allocated to this content of mathematics for the first grade. For this reason, for me it is of scientific interest how this learning content is covered in the new mathematics training kits. Nine first-grade mathematics training sets received approval of the Ministry of Education and Science and of the teachers in Bulgaria. This article is devoted to the issue of clearing the addition and subtraction of numbers up to 20 with the carrying over the ten in the new first grade mathematics kits. I present a comparative analysis of the methodological approaches that the different authors’ groups have used in their textbooks under the “Adding and Subtracting Up to 20 with Carrying over the Ten” section. Interesting results were achieved. The number of lessons foreseen in this section in the different syllabi varies greatly from 20 to 38. When subtracting numbers up to 20 with carrying over the ten, two different approaches are known in the methodology of the mathematics training: subtraction is based on the “subtract a sum from number” ($11 - 4 = ?$ $4 = 1 + 3$; $11 - (1 + 3) = (11 - 1) - 3 = 10 - 3 = 7$; $11 - 4 = 7$) and on the basis of the “subtract a number from sum” ($11 - 4 = ?$ $11 = 10 + 1$; $(10 + 1) - 4 = (10 - 4) + 1 = 6 + 1 = 7$; $11 - 4 = 7$). These two approaches are not represented in all nine textbooks. Explaining how to add and subtract with numbers of up to 20 with carrying over, visualization plays an important role. The best tool to illustrate this is the abacus. Other possible ways to illustrate are schematic models and visualization using the numerical axis. It is best to use all three types of visualization in the lessons of acquiring new knowledge. Some of the training kits presented here lack adequate visualization. This speaks of a lack of a systemic methodical approach in compiling the mathematics training kits. Conclusions are drawn on the advantages and disadvantages of compared textbooks. Conclusions can serve the authors of mathematics training kits in their future work, as well as primary school teachers in choosing training set to teach mathematics.

Keywords: mathematics, first grade, adding and subtracting numbers up to 20, textbooks

1. INTRODUCTION

One of the most important elements of mathematical content in the first grade is the mastering by the students of adding and subtracting numbers up to 20 with carrying over the ten.³⁸ These skills are the basis for addition and subtraction by carrying over the ten with larger numbers. If the students in the first grade do not master to automation all cases of adding and subtracting numbers up to 20 with carrying over the ten, they cannot handle math

³⁸ First Degree Mathematics Curriculum at Email: <http://www.mon.bg/?go=page&pageId=1&subpageId=1699>, last visited on 13.08.2018

in the next classes. Currently, educational reform is being implemented in Bulgaria. It also affects mathematics education in the first grade. 9 training sets of 9 authors' teams are approved by the Ministry of Education and Science. In this article I will present a comparative analysis of the methodological approaches used in these 9 different textbooks. Special attention will be given to the total number of lessons (for new knowledge and exercises) on the subject of “Adding and Subtracting Numbers Up to 20 with Carrying over the Ten” in the mathematics textbooks; in what sequence the different cases of addition and subtraction are studied; whether the two subtraction methods are introduced with a transition to 20. When subtracting the numbers to 20 with carrying over the ten, two different approaches are known in the methodology of the mathematical training: subtraction is based on the “subtracting a sum from number” ($11 - 4 = ?$ $4 = 1 + 3$; $11 - (1 + 3) = (11 - 1) - 3 = 10 - 3 = 7$; $11 - 4 = 7$) and on the basis of the “subtract a number from sum” ($11 - 4 = ?$ $11 = 10 + 1$; $(10 + 1) - 4 = (10 - 4) + 1 = 6 + 1 = 7$; $11 - 4 = 7$). These two approaches are not represented in all nine textbooks. Special attention will also be paid to the visualization used in textbooks.

2. METHODOLOGICAL APPROACHES AND VISUALIZATION IN SECTION “ADDING AND SUBTRACTING NUMBERS UP TO 20 WITH CARRYING OVER THE TEN”

The first textbook to be analysed is Mathematics for First Grade with authors L. Alexieva and M. Kirilova of RIVA Publishing House. This textbook will be quoted further as T1.³⁹ It provides a total of 25 lessons for new knowledge and exercise. Cases of adding and subtracting numbers up to 20 with the carrying over the ten are studied in parallel: one lesson for adding with carrying over and the follow-up lesson on the reverse – subtracting with carrying over. The order is as follows: $9 + 2$; $11 - 2$; $9 + 3$; $12 - 3$; etc. The second way of subtracting – subtracting a number from a sum – is not represented in the textbook at all. The visualization that is used (Picture 1 and Picture 2) includes rows with squares. There is a contradiction in boxing in the action addition and action subtraction. In the action of addition the squares are displayed in separate rows of 10, and in the action of subtraction one line is used with 11 squares. Such visualization has nothing to do with the abacus, which is the most appropriate and natural way to demonstrate to students the addition and subtraction of up to 20 with carrying over. A schematic illustration is also used by presenting the second number as a sum of suitable addends. There is a numerical axis (numeric line) in the textbook, but it does not represent in full the carrying over the ten.



Pic. 1. T1 – addition

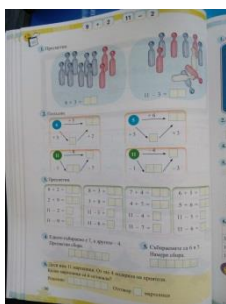


Pic. 2. T1 – subtraction

The second textbook is Mathematics for First Grade, with authors Y. Garcheva and A. Manova of Prosveta Publishing House – Wonderful World.⁴⁰ It has a total of 23 lessons. Here we shall refer this textbook as T2. In this textbook, as well as in T1, the parallel study of the cases of addition and subtraction was adopted: $9 + 2$; $11 - 2$; $9 + 3$; $12 - 3$, etc. The second way of subtracting – subtracting a number from a sum – is not represented in the textbook at all. Illustration used in T2 in the first lesson of new knowledge: $9 + 2$; $11 - 2$ can be seen on Picture 3. Only schematic illustration is used, which is also completely wrong. Instead of representing $6 + 5$ as $6 + (4 + 1)$, it is shown as $6 + (3 + 2)$. In the second illustrated example $5 + 6$, instead of being $5 + (5 + 1)$, it is represented as $5 + (3 + 3)$. This is totally unacceptable from a methodological point of view. In the same task, subtraction with carrying over is represented by correct schematic patterns.

³⁹ Alexieva, L., M. Kirilova. Mathematics for First Grade, RIVA, S., 2016.

⁴⁰ Garcheva, Y., A. Manova. Mathematics for First Grade. Prosveta, S., 2016.



Pic. 3. T2 – addition and subtraction

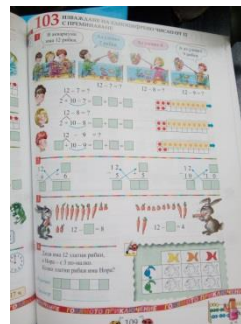
The third textbook is First Class Mathematics with authors V. Angelova and R. Topalski of Prosveta Plus Publishing House.⁴¹ A total of 28 lessons for new knowledge and practice have been developed in the textbook. We shall refer this textbook as T3. The authors introduce the two arithmetic actions – adding and subtracting in sequence. First, all cases of adding up to 20 with carrying over the ten are studied: adding to 9; adding to 8; adding to 7, and so on. In the second part of the section only the subtraction cases are taken: subtraction of 11; subtraction of 12, etc. The second way of subtraction is given a few lessons after the first method of subtraction is introduced and confirmed. In our view, this is a wrong methodical approach, as students are confused by the proposed alternative once they have mastered and consolidated one approach. There is a lack of consistency in the use of the two subtraction approaches: in some topics only the second is offered, others alternate the use of first and second ways in consecutive lessons. Two types of illustration are used – with circles in two rows (similar to abacus) and schematic models (Picture 4, Picture 5 and Picture 6).



Pic. 4. T3 – addition



Pic. 5. T3 – subtraction 1



Pic. 6. T3 – subtraction 2

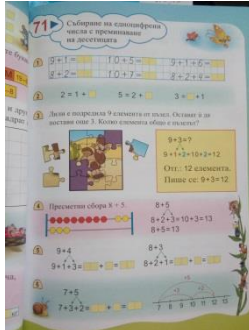
It is noticeable that the schematic modelling of the second subtraction is logically unsettled: the number 12 from which 7 is to be subtracted is represented as $2 + 10$ instead of its natural composition $10 + 2$. This is done for a more convenient presentation of subtraction $10 - 7$, but is methodologically unacceptable.

The fourth textbook is Mathematics for First Grade – Sunshine with authors M. Dimitrova and Tz. Zhekova-Stefanova of Pythagoras Publishers and Golden Duck.⁴² We will refer to this textbook as T4. It features 38 lessons for new knowledge and exercise, which is very good and distinguishes it from T1, T2 and T3. The authors of both T4 and T3 have chosen to introduce the two arithmetic actions consecutively: adding to 9; adding to 8, and so on, and then subtracting from 11; subtraction from 12, etc. The second way of subtraction is introduced in the third lesson as the first and second ways are shown in parallel on the lesson page. The visualization that is used includes 3 types: abacus with 2 rows of balls, schematic modelling, and numerical axis (numerical line). Pictures 8, 9 and 10 show examples of introducing first, second and even third way of subtraction with carrying over the ten. This is a distinctive feature of T4, but it is noticeable that the three types of visualization are not used in the first output task when given the explanations of the new mode of calculation but given in 3 different tasks in the lesson. This is not a good methodical solution. As for the third method of subtraction (by adding the number that we subtract to the number we are subtracting from), this third way does not, in my view, make it easier for the first grade students to

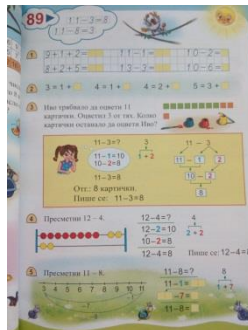
⁴¹ Angelova, B, R. Topalska. Mathematics for First Grade. Prosveta Plus, S., 2016.

⁴² Dimitrova, M., Tz. Zhekova-Stefanova. Mathematics for First Grade. Pythagoras, Golden Duck, S., 2016.

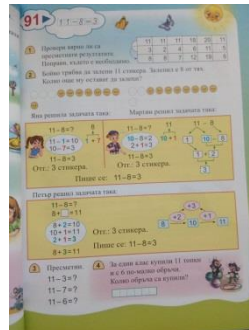
make the calculations. It is only offered in the sixteenth lesson of subtraction with carrying over, which will inevitably lead to confusion for the young pupils. It should be noted that in 5 of the lessons in this section none of the modes of calculation are shown, and the respective illustration used. There is no system in the textbook.



Pic. 7. T4 – addition



Pic. 8. T4 – subtraction 1

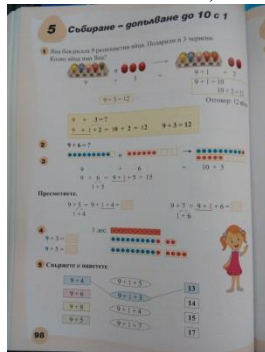


Pic. 9. T4 – subtraction 2

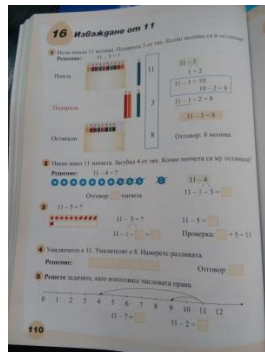


Pic. 10. T4 – subtraction 3

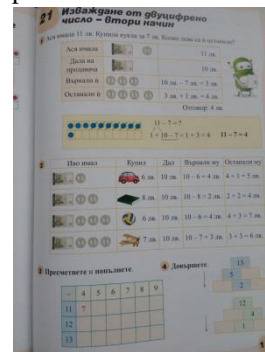
The fifth textbook is Mathematics for First grade with authors Paskaleva, M. Alashka, Z. Lalchev and M. Varbanova of Archimedes Publishing House.⁴³ We will refer this textbook as T5. It provides only 20 lessons to study adding and subtracting numbers up to 20 by carrying over the ten. This is extremely inadequate. Adding and subtracting tasks with the number of up to 20 with carrying over the ten are a total of 72 – 36 of addition and 36 of subtraction. All of them must be mastered by first-graders to automatism. It is impossible to do this with 20, 23 or 25 lessons. Actions adding and subtracting up to 20 are studied in sequence. The first and second ways of subtraction are introduced in this textbook. Unfortunately, the second way is offered to students only in the seventh lesson of subtraction when it is supposed that the children have mastered and confirmed the first way. Such a late introduction of a second way of subtraction can only confuse and hinder first-graders. The visualization used in T5 is schematic modelling and circles, but in one of the tasks the circles are located on two rows and resemble abacus, whereas in another task in the same lesson, the circles are on one line and have nothing to do with the abacus. There is no system and a clear concept of visualization. The schematic depiction of action used is completely unacceptable as an image. In both T5 and T3, when introducing the second subtraction method, the representation of the number 11 is used, from which we subtract, as 1 + 10 rather than 10 + 1. The illustration is represented in Pictures 11, 12 and 13.



Pic. 11. T5 – addition



Pic. 12. T5 – subtraction 1



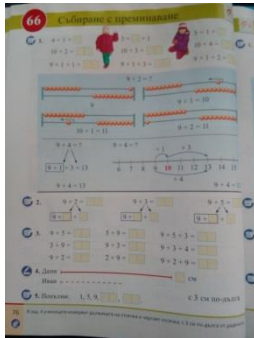
Pic. 13. T5 – subtraction 2

The sixth textbook is Mathematics for First Grade with authors T. Vitanov, G. Kirova, S. Sharkova, I. Pushkarova and D. Parusheva of Anubis Publishers.⁴⁴ This textbook will be referred to as T6. It provides 31 lessons for new knowledge and exercise. The actions of addition and subtraction are studied in sequence. The layout is 9 +; 8 + and so on, and 11 -; 12 - and so on. The second method of subtraction is introduced, which is done in the second lesson. This is a good methodical solution as students can immediately benefit from the two approaches proposed and choose the one that is easier for them. All three types of visualization are used in the illustrations: abacus with 2

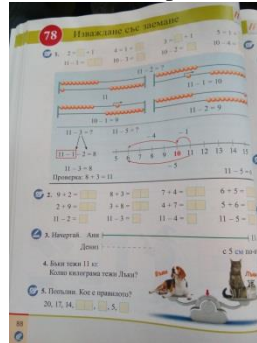
⁴³ Paskaleva, Z., M. Alashka, Z. Lalchev, M. Varbanova. Mathematics for First Grade. Archimed, S., 2016.

⁴⁴ Vitanov, T., G. Kirova, Z. Sharkova, I. Pushkarova, D. Parusheva. Mathematics for First Grade. ANUBIS, S., 2016.

rows of balls, schematic modelling and numerical axis (numerical line). All three types of illustration are applied in the new knowledge box, as seen in Pictures 14, 15 and 16. Only in the T6 the abacus is shown in real form and by successive frames, clearly illustrating the displacement (manipulation) of the abacus balls.



Pic. 14. T6 – addition

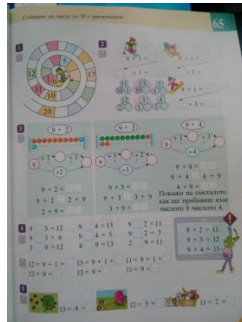


Pic. 15. T6 – subtraction 1



Pic. 16. T6 – subtraction 2

The seventh textbook is Mathematics for First Grade with authors M. Bogdanova and M. Temnikova of BULVEST 2000 Publishing House.⁴⁵ Here I will refer the textbook as T7. Lessons foreseen in it are 38 for new knowledge and for exercise. This is an excellent methodical solution. The actions of addition and subtraction are studied in sequence. The second way of subtraction is shown, but yet in the first lesson, which is not a good methodical solution. Students should have a chance to know the first way of subtraction, albeit briefly. A serious drawback of T7 is that new cases of addition and subtraction with carrying over the ten are introduced in exercise lessons. The visualization used by the authors is circles (in two rows of 10), schematic modelling, and sticks to illustrate the second way of subtraction (Picture 17 and Picture 18). It should be emphasized that the use of stick image for this second subtraction method as done in T7 is absolutely insubstantial. For Task 11 - 9, the number 11 should be visualized with 1 tenth, tied in a tuft and 1 single stick. Then it is natural to separate the single stick first ($11 - 1 = 10$), then loosen the bundle and remove 8 more sticks ($10 - 8 = 2$), finally seeing how many sticks remain ($11 - 9 = 2$). Instead, the authors gave an image of 11 free sticks, one of which is on the right and separated by a line and outlined 9 of the ten sticks on the left. It can be said that such visualization is completely inadequate.



Pic. 17. T7 – addition



Pic. 18. T7 – subtraction 1 and 2

The eighth textbook is Mathematics for First Grade with authors R. Petrova, R. Stoyanova and P. Daskova of SCORPIO Publishing House.⁴⁶ This textbook will be referred to as T8. The lessons for new knowledge and exercise are 33. Adding and subtracting up to 20 with carrying over the ten are studied in parallel – 2 lessons for addition together with 2 lessons for subtraction and joint exercises afterwards. Two ways of subtraction are introduced, but methodically wrong the first way is called second and vice versa. Both ways are shown in the fourth lesson, which is not a good methodical solution. In the 21st and 22nd lessons from the section, the two modes change places and are rightly marked as the first and second way. After a few lessons, the two ways are replaced again. At the end of the section in the lessons for new knowledge, the cases of subtraction are only explained in the first way. It can be summed up that there is an extremely improper methodical approach with a lack of system and consistency.

⁴⁵ Bogdanova, M., M. Temnikova. Mathematics for First Grade, BULVEST 2000, S., 2016.

⁴⁶ Petrova, R., R. Stoyanova, P. Daskova. Mathematics for First Grade. SCORPIO, S., 2016.

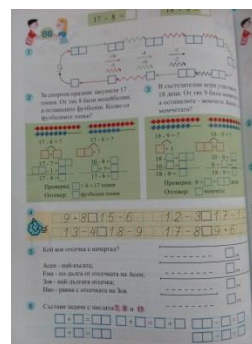
Illustrations are circles (in the addition) and circles in two rows (in the subtraction), as well as schematic patterns, which can be seen in Pictures 19, 20 and 21.



Pic. 19. T8 – addition

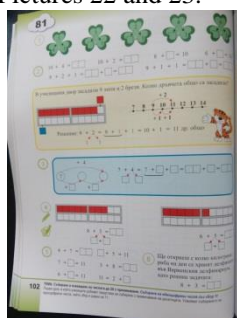


Pic. 20. T8 – subtraction 1

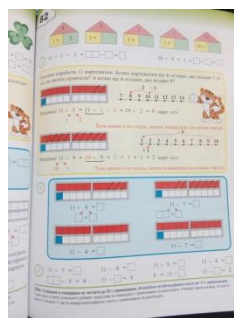


Pic. 21. T8 – subtraction 2

The last, ninth textbook is Mathematics for First Class with authors T. Valkova, T. Momcheva, R. Miteva, V. Damaskova, D. Stoyanova, D. Dimitrova and Iv. Dimitrova of Life and Technique Publishing House.⁴⁷ We will refer to this textbook as T9. A total of 26 lessons for new knowledge and exercise are offered in the textbook. The addition and subtraction actions are introduced in parallel, the chosen approach being: sum 11 and subtraction of 11 ($9 + 2$, $8 + 3$, $7 + 4$ etc and $11 - 2$, $11 - 3$, etc.). This methodical approach has long been used in Bulgarian mathematical textbooks and has shown its inappropriateness. From all analysed textbooks, we find this approach only in T9. The second way of subtraction is shown in the first lesson of new knowledge simultaneously with the first way. As explained above in the article, it is not appropriate for first-class students to have two different approaches at the same time. The visualization used by the authors of T9 is squares, numerical axis, and schematic modelling. A good solution is to apply the three types of visualization to the new knowledge box when solving the output task, as shown in Pictures 22 and 23.



Pic. 22. T9 – addition



Pic. 23. T9 – subtraction 1 and 2

3. RESULTS OF THE COMPARATIVE ANALYSIS OF THE BULGARIAN MATHEMATICS TEXTBOOKS FOR THE FIRST GRADE

The summarized results of the comparative study of the nine first-grade mathematics textbooks in Bulgaria are presented in Table 1. The textbooks are compared to several important criteria in the methodological development of the topics in the section “Adding and Subtracting Numbers Up to 20 with Carrying Over the Ten.” First criterion is the number of lessons in the section. To master this important first-degree content, a maximum of lessons are needed for new knowledge and practice. The second criterion is the chosen methodological approach for introducing addition and subtraction cases. There are three options: parallel, consecutive and simultaneous. In more textbooks, sequential study has been chosen and this is methodologically justifiable. The next criterion when comparing textbooks is whether a second method of subtraction is introduced. Illustration with its varieties is also subject to analysis. Table 1 reveals the advantages and disadvantages of individual first-grade mathematics textbooks.

⁴⁷ Valkova, T. and colleagues. Mathematics for First Grade, Life and Technique, Varna, 2016.

Textbook	Lessons (number)	Method of introducing	Second way of subtraction	Abacus/circles/squares	Schemes	Numerical axis
T1	25	parallel	-	-	+	-
T2	23	parallel	-	-	+(wrong)	-
T3	28	consecutive	+	+	+	-
T4	38	consecutive	+	+	+	+
T5	20	consecutive	+	-	+	-
T6	31	consecutive	+	+	+	+
T7	38	consecutive	+	+	+	-
T8	33	parallel	+	+	+	-
T9	26	simultaneous	+	+	+	+

Table 1. Comparative Analysis of Components in the Nine Textbooks of Mathematics for First Grade

With regard to the number of lessons in the section “Adding and Subtracting Numbers Up to 20 By Carrying over”, some textbooks (T5, T2, T1, T3, T9) are unacceptable due to the extremely low number of lessons in this important section. A good methodical solution is that the number of lessons is over 30 (T6, T8, T4 and T7). The methodical order of the topics in the section is three types: parallel (T1, T2 and T8), simultaneous (T9) and consecutive (T3, T4, T5, T6 and T7). With the purpose of mastering and automating the knowledge of adding and subtracting up to 20, consecutive learning has its advantages. Subtracting up to 20 with carrying over can be done in two different ways. A good methodical solution is these two ways to be presented to students and give them the opportunity to choose one, which is easier for them. In this regard, two of the textbooks analysed here are distinguished by this disadvantage that they do not offer the second subtraction method at all (T1, T2). All other textbooks offer students both ways of subtraction. But this should not be done yet in the first lesson (T7, T9), because students have not yet fully perceived and understood the first way. The most appropriate is the decision to show the second way of subtraction in the second lesson (T6). The introduction of the second way later (T3, T4, T5, T8) is methodologically erroneous as students have already mastered and validated the calculation in the first way and the introduction of a new, different way of subtraction later on can only lead to confusion. Regarding the visualization in the analysed textbooks, one can say that the best methodological solution is to use all three types of visualization: through abacus, through schemes and by a numerical axis (numerical line). Such visualization exists only in three of the textbooks (T4, T6, T9). There is illustration with abacus only in T6, with separate images showing the two steps, the two moves of the balls on the abacus. Many textbooks use circles or squares, which is relatively acceptable only if the circles (squares) are in 2 rows of 10 (T3, T4, T7, T9). In the other textbooks, it is not possible to talk about illustration of the abacus type (T1, T2, T5, T8). Illustration with schematic models exists in all textbooks analysed. The numerical axis (numeric line) is a great visual aid, on which two arrows help illustrate the two steps when numbers up to 20 are added and subtracted with carrying over the ten. Such visualization is only used in textbooks T4, T6 and T9. It can be said that the best visualization is in the textbooks T4, T6 and T9.

4. CONCLUSION

From the comparative analysis of the curriculum content in the section “Adding and Subtracting Numbers Up to 20 with Carrying over the Ten”, between the nine mathematics textbooks in force in 2016/2017, it can be concluded that in some of them there are serious shortcomings – insufficient number of lessons provided for new knowledge and exercise, methodologically unacceptable approach to introducing addition and subtraction, lack of the second way of subtraction, inadequate visualization and even wrong illustration. Our aim is to show the good aspects and weaknesses of the analysed textbooks to enrich the methodological literature and to help the initial teachers in Bulgaria to make reasoned choice of textbook to work with their first-grade students during the academic year 2018/2019.

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