NON UNIFORM ARRAY ANTENNA WITH THREE LUNEBURG LENSES

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Abstract: A fundamental part of the work of any communication device, such as radio, television, mobile phone, etc., is the signal filtration. Filtration is the process of separation of the signals that carry the needed information, from other signals, noise, etc. This is necessary for the proper operation of the communication device. The filters synthesis is a mathematical problem for an approximation of the ideal filter response. This is an ideal function, comprising rectangular shape. The ideal function cannot be realized because it contradicts the basic physical principles. For this reason, it is replaced by another, which can be realized by the existing technical devices. In many filter syntheses problems, the approximating function is polynomial. The goal is to define a low degree polynomial that approximates the ideal function with high precision (minimal error). A new mathematical method in the approximation theory is named „compressed cosines ". The method is applicable in the communication technologies. The method uses Remez' exchange algorithm, which has fast convergence and low computational complexity. As a result, an optimal third degree trigonometric polynomial is defined. The proposed polynomial meets the requirements of the approximation theory: approximation of the ideal filter function with high precision (minimal error). The described properties are proved by comparison with Chebyshev polynomials. In the approximation theory such polynomial of 4-th degree (practically 3-th degree) with better approximating properties is not known. The proposed polynomial is successfully applied for digital FIR filters and antenna arrays design**.** This article is an extension of research on the method of compressed cosines in array antennas synthesis. Theory and technical solution for the synthesis of non-uniformly spaced linear array with three Luneburg lenses are proposed. The applied theory allows synthesis of broadband array with high selectivity and without side-lobe array factor. Such an antenna system has no analogues. The array antenna is suitable for radio astronomy and radio location.
Keywords: array antenna, array factor, polynomial, approximation

1. INTRODUCION

Uniform linear array (ULA) antennas are spatial filters. The theory assumes that they consist of N isotropic radiators, evenly spaced at distance d along the axis z. Isotropic radiator have no selective properties. The radiation diagram is formed by spatial function array factor (AF) which has the form of $\sin x/x$ function. The diagram pattern has a main lobe perpendicular to the z axis and declining side lobes that depend on the number of radiators. ULA with a larger number of radiators have a better spatial selectivity. In practical realization it is not always possible for the radiators to be located at equal distances. In this regard, since the 60s of last century, several researchers developed a theory of Non Uniform Linear Array (NULA) Antenna [1]. The studies [2, 3] are limited to determining the distances between the radiators to achieve a desired pattern that is equal to or similar to that of an ULA with an equal number of radiators. For the purpose of synthesis, the iteration algorithms are used, with the number of array elements N being greater than nine. It is natural that with a small number of emitters, for example $N = 3$, high selectivity cannot be achieved. Figure 1 shows a non-uniform linear antenna array with 3 radiators.

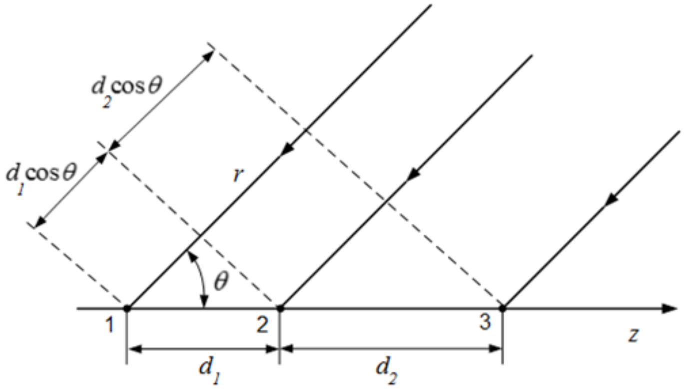

Fig. 1. Three-element NULA

In the picture θ is the azimuthal angle and the distances $d_1 \cos\theta$ and $d_2 \cos\theta$ are proportional to the signal delay time between two adjacent radiators. Different distances between elements aggravate the selective properties of the antenna grid. This is demonstrated in *Figure 2*, where ULA and NULA directional diagrams are shown at $N = 3$.

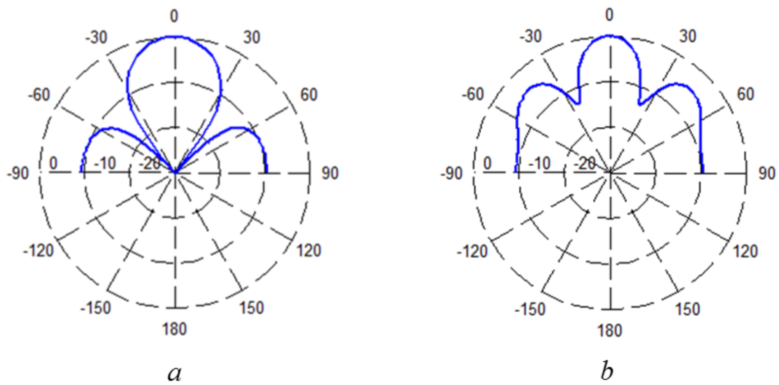

Fig. 2. Three-element ULA, $d = \lambda/2$ (a). Three-element NULAA, $d_1 = \lambda/2$; $d_2 = \lambda$ (b)

On the other hand, the small number of elements does not allow the successful implementation of iterative algorithms to determine optimal distances between the radiators. This article proposes NULA, which resolved these design problems.

2. THEORY OF THE METHOD

In [4] is described ULA antenna with three Luneburg lenses. The theory is based on a new approximation method with "*compressed cosines*" [5]. The proposed array antenna array can theoretically work across the entire radio diapason with equal array factor.

$$A(\theta) = \left|1 - 2\exp\left(-j2\varphi(\theta)\right) + 1\exp\left(-j4\varphi(\theta)\right)\right|. \quad (1)$$

In (1) the coefficients 1, -2, 1 are the amplitudes, and the exponents the phases of the antenna currents. The phases are expressed by the non-linear function

$$\varphi(\theta) = -\frac{\pi}{2}\left(\operatorname{erf}\left(\beta kd \sin\theta\right) + 1\right), \quad (2)$$

where $\operatorname{erf}(.)$ is the integral Gaussian error function, $\theta \in [-\pi/2\,, \pi/2]$ is the azimuth spatial angle, and $k = 2\pi/\lambda$ is a phase constant. The parameter $\beta > 0$ is determined by the half power beam width (-3 dB) $\Delta\theta_{HPBW}$

$$\beta = \frac{\operatorname{erf}^{-1}\left(\frac{1}{\pi}\arccos\left(1 - \sqrt{2}\right)\right)}{kd\sin\left(\Delta\theta_{HPBW}\right)}, \quad (3)$$

where $\operatorname{erf}^{-1}(.)$ is the inverse integral Gaussian error function.

The array factor has no side lobes; the parameter β changes the beam width of the diagram - *Figure 3*.

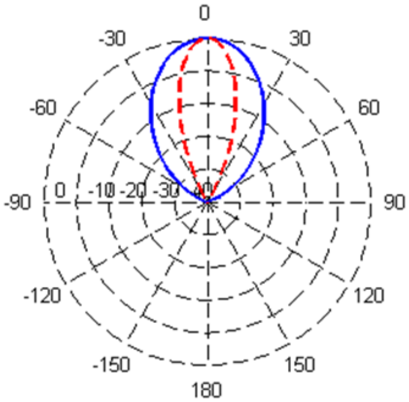

Fig.3. Array factor. Continuous line $\beta = 0.5$; dashed line $\beta = 1$

3. REALIZATION OF NON UNIFORM LINEAR ARRAY ANTENNA WITH THREE LUNEBURG LENSES

The task is to implement the array factor (1) using technical devices, provided $d_1 \neq d_2$ – *Figure 1*. The ULA properties described in the previous paragraph are due to two technical solutions: the use of Luneburg lenses and the realization of the array factor by digital signal processing in the time domain.

The Luneburg lens is a *frequency independent* isotropic sphere because its focusing properties depend entirely on the dielectric permeability gradient. The Antenna lens focuses flat electromagnetic waves (EMWs) from all directions of the azimuthal angle θ at points that form a semicircle at the diametrically opposed end of the lens. For example, in *Figure 4*, the rays of the flat EMW of $\theta \approx -30^0$ direction are focused at point *a*, and the rays of $\theta \approx 45^0$ direction are focused at point *b*.

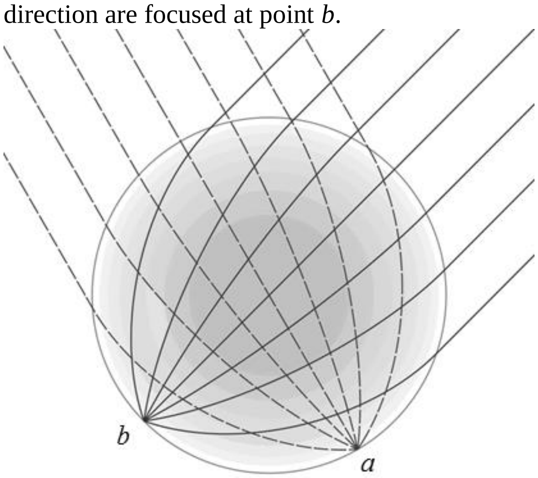

Fig. 4. Luneburg lens

The Lundeberg's lens with zero diameter is an isotropic point radiator. This solution makes the array antenna broadband.

As it is mentioned above, the different distances d_1 and d_2 between the radiators change the selectivity of the grid multiplier (*Figure 2*). For the realization a second technical solution is proposed - phase alignment and application of the temporal weighting function.

The signals are received from *N* equally spaced points located on the focal semicircle that correspond to *N* azimuthal directions. The received analog signals are converted, discredited, delayed and summed up - *Figure 5*.

The phase alignment is done with delayed lines whose time delays are determined by the dependencies:

$$t_{A1}(\theta)=\begin{cases}0, & \theta\in[-\pi/2,0]\\ \dfrac{d_1+d_2}{c}\sin\theta, & \theta\in[0,\pi/2]\end{cases}; \qquad (4)$$

$$t_{A2}(\theta)=\begin{cases}\dfrac{d_1}{c}|\sin\theta|, & \theta\in[-\pi/2,0]\\ \dfrac{d_2}{c}\sin\theta, & \theta\in[0,\pi/2]\end{cases}; \qquad (5)$$

$$t_{A3}(\theta)=\begin{cases}\dfrac{d_1+d_2}{c}|\sin\theta|, & \theta\in[-\pi/2,0]\\ 0, & \theta\in[0,\pi/2]\end{cases}, \qquad (6)$$

where c is the speed of light in vacuum.

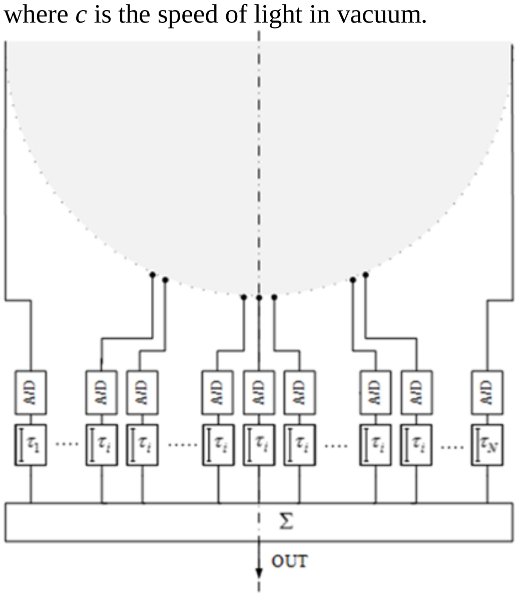

Fig. 5. Lower hemi sphere of Luneburg's lens with digital signal processing

Figure 6 shows the time delays for the three lenses at $d_1=\lambda/2$, $d_2=\lambda$, normalized relative to the nominal time t_{nom}.

$$t_{nom}=\lambda/(2c)=1/(2f)\,. \qquad (7)$$

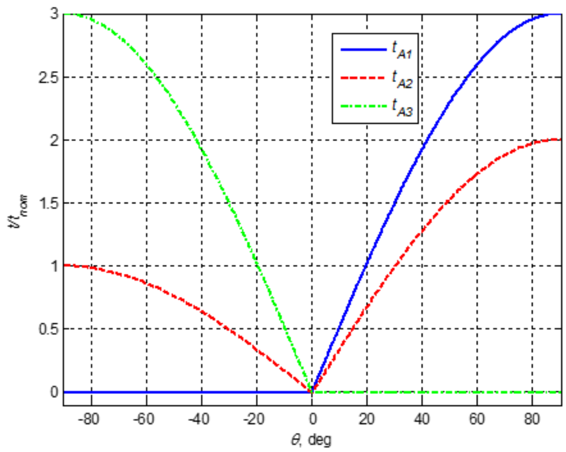

Fig. 6. Time delays for phase alignment

The flat electromagnetic wave front is a virtual plane in which the electromagnetic wave is with an equal phase. The application of time delays for flat EMWs have the physical sense of converting the focus semicircle into a straight line on which the EMWs fronts of all *N* azimuth directions become parallel to it and therefore arrive at the same phases. This technical solution eliminates the negative effect of the difference in distance between the three lenses d_1 and d_2, and they can also be arbitrary. It follows that at fixed distances d_1 and d_2, the antenna grid can operate simultaneously at one or more arbitrary frequencies similar to filter banks in the spatial domain.

After phase alignment, a weight function is added to the delays of the second and third lenses (*Figure 7*).

$$w(\theta) = t_{nom}\left(1 - \frac{\varphi'(\theta)}{2\beta\pi\sqrt{\pi}}\right) = \frac{1}{2f}\left\{1 - \frac{\cos\theta}{\exp\left[(2\beta\pi\sin\theta)^2\right]}\right\}, \tag{8}$$

where

$$\varphi'(\theta) = \frac{d\varphi}{d\theta} = \frac{2\beta\pi\sqrt{\pi}\cos\theta}{\exp\left[(2\beta\pi\sin\theta)^2\right]} \tag{9}$$

is the derivative of the phase function (2) for $d = \lambda/2$. Parameter β is calculated from (3) under the same condition: $d = \lambda/2$

$$\beta = \frac{\text{erf}^{-1}\left(\frac{1}{\pi}\arccos\left(1-\sqrt{2}\right)\right)}{\pi\sin\left(\Delta\theta_{HPBW}\right)} \approx \frac{0.2043}{\sin\left(\Delta\theta_{HPBW}\right)}. \tag{10}$$

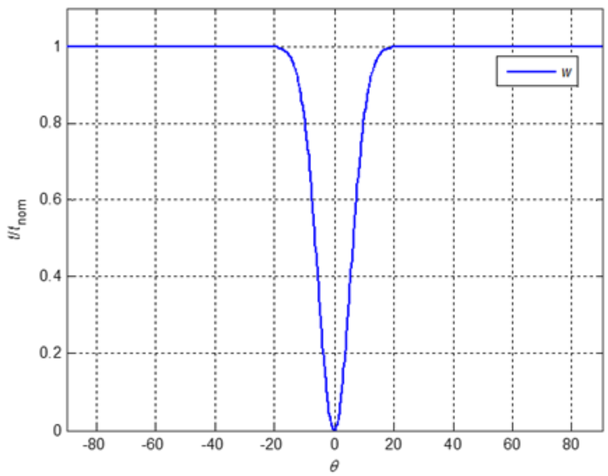

Fig. 7. Weighting function, $\Delta\theta_{HPBW} = 10^0$

This technical solution realized phase multipliers in the second and third terms of the array factor (1) in the time area.

Thus, the overall delays for the three lenses are as follows:

$$\tau_{A1}(\theta) = t_{A1}(\theta); \quad (11)$$

$$\tau_{A2}(\theta) = t_{A2}(\theta) + w(\theta); \quad (12)$$

$$\tau_{A3}(\theta) = t_{A3}(\theta) + 2w(\theta). \quad (13)$$

Figure 8 shows the time delays of the three lenses for $\Delta\theta_{HPBW} = 10^0$ и $d_1 = \lambda/2$, $d_2 = \lambda$.

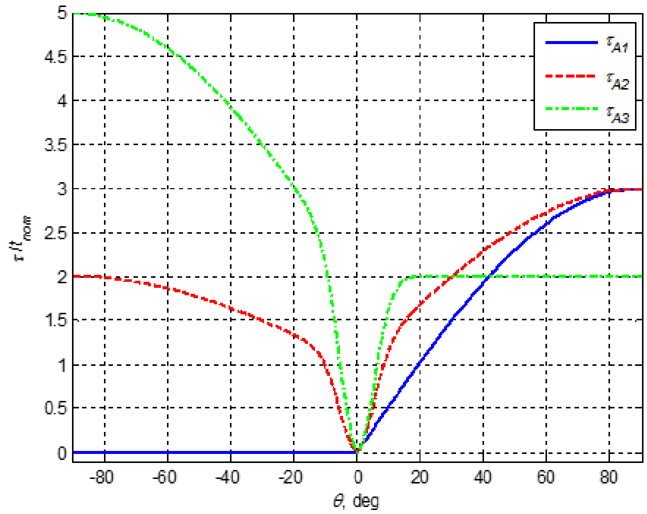

Fig. 8. Delay times of the three Luneburg's $\Delta\theta_{HPBW} = 10^0$

Figure 9 shows the diagram of the array antenna.

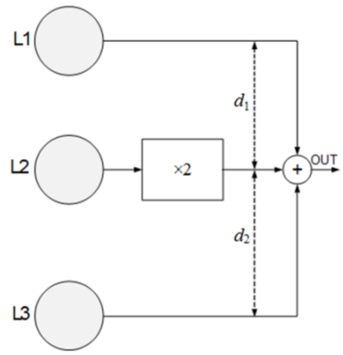

Fig.9. NULA antenna structure with 3 Luneburg lenses

The cumulative signal from the second lens is multiplied by 2, according to the coefficient of the second term of the array factor (1). At the output, all signals are summed.

The described theory has been demonstrated by a Matlab computer simulation for a 3-element antenna array with distances $d_1 = 150$m ($\lambda/2$ to1MHz), $d_1 = 300$m and a targeting beam width $\Delta\theta_{HPBW} = 10^0$. For frequencies 300 kHz; 1MHz and 3MHz are used 91×3 sinewave files, with a sampling frequency of 200MHz. The sinusoids are phased so as to simulate the reception of signals at three isotropic radiators from 91 spatial directions of the angle. The signal processing uses equations (4-13) the cumulative signal from the second antenna is multiplied by two.

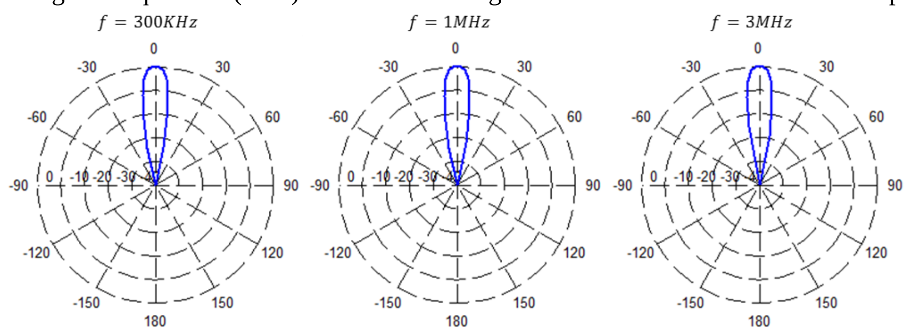

Fig. 10. Simulation of Matlab

4. CONCLUSION

A theoretical method and technical solution for the construction of an ultra-wideband three element NULAA with Luneburg lenses is proposed. The processing of the signal formulas (4-13) allows the creation of a prototype for filter banks in spatial signal filtration - *Fig. 8*. The proposed method and array antenna have no analogue in the domain of antenna technique.

REFERENCES

[1] Schjaer-Jacobsen, H., K. Madsen, Synthesis of Nonunifomly Spaced Arrays Using a General Nonlinear Minimax Optimization Method, IEEE Transactions on Antennas and Propagation, Volume: 24, Issue: 4, Jul 1976, pp. 501-506.

[2] Preetham, B. Kumar, G. R. Branner. Design of Unequally Spaced Arrays for Performance Improvement, IEEE Transactions on Antennas and Propagation, vol. 47, no. 3, march 1999, pp. 511-523.

[3] Oraizi, H. and M. Fallahpour. Nonuniformly Spaced Linear Array Design for the Specified Beamwidth/Sidelobe Level or Specified Directivity/Sidelobe Level with Coupling Considerations, Progress in Electromagnetics Research M, Vol. 4, 2008, pp. 185–209.

[4] Apostolov, P. Three-element broadband narrow beam without side lobes array antenna, IET Microwaves, Antennas & Propagation, Vol: 10, Issue: 11, 2016, pp. 1212 - 1217.

[5] Apostolov, P. Linear Equidistant Antenna Array with Improved Selectivity, IEEE Transaction on Antennas and propagation, Vol.59, Issue10, Aug. 2011, pp. 3940-3943.