# THE GEOMETRIC EDUCATION CONTENTS THE NEW STUDY SETS OF MATHEMATICS FOR FIRST GRADE 

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#### Abstract

Currently, educational reform takes place in Bulgaria. It also concerns the curriculum of mathematics in the first grades. The first-grade mathematics curriculum was approved by Order No. РД 09-1857, dated December 17, 2015. It came into force in the school year 2016/2017. Unfortunately, the study was done with the old mathematics study sets. One of the fields of mathematical competence in the curriculum for the first grade is "Geometric figures and bodies". Geometric knowledge is intended to be studied in unity with the arithmetic and knowledge of the units of measurement. Students of the first grade should deepen their insights into the geometric figures: segment, triangle, square, and rectangle. They should be able to draw a segment of a given length, measure geometry figures and segments in centimetres, draw geometric shapes on a square mesh, know the geometry figures elements - sides and apexes. Through the study in geometry, students of first grade should develop logical thinking, power of observation, imagination, creativity and others. During the school year 2017/2018, Bulgarian first grade teachers will have the opportunity to teach with their chosen study sets for the first time. Approved by the Ministry of Education and Science and by the teachers, nine sets of mathematics for the first grade were adopted. The annual measurement of learning outcomes at the end of the initial stage of the primary education, the so-called national external evaluation, traditionally reveals two tasks: mastering geometric content and working with text problems. For this reason, it is a scientific interest for me of how this learning content is covered in the new mathematics study sets. This article is devoted to the question of geometric content in new study sets in mathematics for the first-grade. I present a comparative analysis of the geometric problems in textbooks and notebooks to them. All problems and exercises with geometric content are systematized and classified. The tables present the relative share of each type of geometric problems in individual textbooks. Interesting results were achieved. Geometric problems and exercises are of the following types: 1) Recognition (naming, definition of species); 2) Counting in combination figures; 3) Drawing in a square mesh (point matching); 4) Colouring; 5) Measuring of segments and sides of geometric shapes; 6) Measurement of objects; 7) Drawing of segments and geometric shapes; 8) Symmetry; 8) Text problems with geometric learning content. The number of geometric problems in the different study sets varies considerably from 149 to 54 . In some of the study sets, geometric problems are totally inadequate in number. There are separate types of geometric problems that are completely absent in some of the analyzed study sets. How can students be able to solve a certain type of geometric problems if this type of task occurs only once in a study set? There was an imbalance in the number of geometric problems in the individual notebooks to the first grade mathematics textbooks. This speaks of a lack of systematic methodical approach in the writing of mathematics study sets. Conclusions are drawn on the advantages and disadvantages of compared textbooks. Conclusions can be used by the authors of second, third and fourth grade mathematics study sets.


Keywords: geometry, first grade, mathematics, textbook.

## 1. INTRODUCTION

Historically, mathematics education has changed its dimensions and forms as a result of the changing needs of society over the centuries and the age-specific features of the different generations to reach its present appearance. This applies in particular to geometry. The term "geometry" means earthmeasuring (from Greek $\gamma \dot{\eta}$-land and $\mu \varepsilon \tau \rho \varepsilon ́ \omega$-measure). From the excerpts of preserved ancient Egyptian writings, it has been found that geometry has evolved not only from the need to measure the earth, but also from measurements of surfaces and volumes in earthworks and civil works. [26, p. 8]
According to Walle (2001), five levels of thinking can be distinguished in geometry: 1) On the first level, the geometric figures differ in their form without analyzing their properties; 2) On the second, the geometric figures are analyzed and their properties are established; 3) On the third, the logical arrangement of the geometric figures and their properties is made; 4) On the fourth, deduction is considered as a way of building and developing the whole geometric theory; 5) On the fifth, geometrical theory develops as an abstract deductive system. In elementary school the study of the geometric material takes place mainly at the first and second level. [35, p. 27]

## 2. GEOMETRY IN THE FIRST GRADE

Geometric content is an indispensable part of the mathematics education in the first grade in Bulgaria. Students yet from the first grade get acquainted with the basic geometric figures and shapes, with some of their features, build skills for measuring and drawing, solve text problems with geometric learning content, learn to find circumference and surface of a geometric figure. In the first grade mathematics curriculum, approved by Order No РД 09-1857 of 17.12.2015, the new knowledge and skills in geometry, which are to be assimilated in the first grade, are defined by the new concepts and the expected results such as: deepening of the ideas of the geometric shapes of segment, triangle, square, rectangle, and build initial skills for drawing geometric shapes into a square mesh. [34] Geometric knowledge and skills contribute to the general mathematical development of students. They contribute to the development of observation, logical and spatial thinking, imagination, constructive abilities, wits and creativity in children. Geometry study promotes the aesthetic upbringing of students. The precision of the execution of the drawings, as well as the geometric problems related to beautiful ornaments, embroidery, symmetry, helps to create aesthetic taste in children. [11, p. 184] At the national external evaluation at the end of the fourth grade, it has been established for years that students show lesser results in geometric problems as well as in text problems. This provoked me to explore the geometric content of the new mathematics study sets for the first grade, which will be used in the Bulgarian schools from the academic year 2017/2018. The complete study sets - a textbook and notebooks of nine authors' collections - are under analysis. Results have been considered and conclusions were made.

## 3. GEOMETRICAL PROBLEMS IN THE STUDY SETA OF MATHEMATICS FOR THE FIRST GRADE

I will consecutively present the problems and exercises with geometric content in the nine approved mathematics study sets for first grade: ANUBIS Publishing House - authors T. Vitanov, G. Kirova, Z. Sharkova, I. Pushkarova, D. Parusheva [12] [13] 14]. In the study for short, the textbook will be referred to as T1; RIVA - authors L. Aleksieva and M. Kirilova (authors of the notebooks L. Alexieva and E. Angelova-Ananieva [1] [2] [3]. In the study, the textbook will be referred to as T2; Prosveta Plus - authors V. Angelova and R. Topalski [4] [5] [6]. In the study, the textbook will be referred to as T3. Prosveta - authors Y. Garcheva and A. Manova [19] [20] [21]. In the study, the textbook will be referred to as T4. Pythagoras and Golden Duck Publishers - authors M. Dimitrova and Tz. Zhekova-Stefanova [22] [23] [24] [25]. In the study, the textbook will be referred to as T5. Archimedes authors Z. Paskaleva, M. Alashka, Z. Lalchev and M. Varbanova [27] [28] [29]. In the study, the textbook will be referred to as T6. SCORPIO - authors R. Petrova, R. Stoyanova, P. Daskova [30] [31] [32] [33]. In the study, the textbook will be referred to as T7. BULVEST 2000 - authors M. Bogdanova and M. Temnikova [7] [8] [9] [10]. In the study, the textbook will be referred to as T8. Life and Technique - authors T. Valkova, T. Momcheva, D. Stoyanova, R. Miteva, D. Dimitrova, V. Damaskova, I. Dimitrova [15] [16] [17] [18]. In the study, the textbook will be referred to as T9.
The following types of geometric problems and exercises are classified in the content-analysis of the compared study sets (textbook and notebooks thereto): 1) Recognition (naming, definition of the type); 2) Counting in combined figures (and dividing the figure into geometric shapes); 3) Drawing of segments and geometric shapes; 4) Measurement of geometric segments and sides; 5) Drawing in square mesh (and linking points to forming a figure); 6) Measuring the length of depicted objects; 7) Problems of symmetry; 8) Colouring of geometric shapes; 9) Text problems with geometric content. In the text below, the different types of geometric exercises will be indicated with their quantity and their respective number.

## 1. Study set T1

The total number of geometric problems in the textbook is 91 . They are all of the types listed above. In the study notebook No. 1, the geometric problems are 22 and in the study notebook No. 2 they are 36.

Table 1. Geometric content in T1

| Types | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| T1 | 13 | 26 | 15 | 15 | 8 | 2 | 3 | 3 | 6 |
| Notebook <br> No. 1 | 3 | 6 | 7 | 2 | 3 | 1 | 0 | 0 | 0 |
| Notebook <br> No. 2 | 0 | 9 | 4 | 14 | 3 | 1 | 3 | 0 | 2 |

Total of the set of problems is 149 . This is the study set containing the largest number of geometric problems and exercises. It can be said that there are relatively few problems of colouring (8) and measuring lengths of depicted objects (6).

## 2. Study set T2

The total number of geometric problems in the textbook is 59 . They are of 8 types. In study notebook No 1 the geometric problems are 30 and in the study notebook No. 2 they are 31 . Total of the set of problems is 120. A significant drawback of this study set is the complete absence of text problems with geometric learning content.

Table 2. Geometric content in T2

| Types | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| T2 | 7 | 14 | 6 | 13 | 13 | 3 | 1 | 2 | 0 |
| Notebook <br> No. 1 | 4 | 8 | 3 | 1 | 2 | 3 | 9 | 0 | 0 |
| Notebook <br> No. 2 | 2 | 6 | 5 | 7 | 5 | 3 | 3 | 0 | 0 |

3. Study set T3

The total number of geometric problems in the textbook is 58. They are of 8 types. In notebook No. 1 the geometric problems are 9 and in the study notebook No. 2 they are 29 . The total number of problems in the set is 96 . The imbalance between the number of geometric problems in the two study notebooks is impressive. In Notebook 1, geometric problems are only of two types.

Table 3. Geometric content in T3

| Types | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| T3 | 8 | 4 | 10 | 13 | 9 | 2 | 6 | 0 | 5 |
| Notebook <br> No. 1 | 2 | 0 | 0 | 0 | 0 | 0 | 7 | 0 | 0 |
| Notebook <br> No. 2 | 5 | 2 | 6 | 6 | 3 | 1 | 0 | 5 | 1 |

## 4. Study set T4

The total number of geometric problems in the textbook is 70 . They are of 8 types. In study notebook No. 1 the geometric problems are 35 and in the study notebook No. 2 they are 29 . The total number of problems in the set is 134. In this study set there are completely absent text problems with geometric learning content, which is a disadvantage.

## Table 4. Geometric content in T4

| Types | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| T4 | 15 | 6 | 13 | 11 | 9 | 5 | 6 | 5 | 0 |
| Notebook <br> No. 1 | 3 | 4 | 7 | 4 | 4 | 2 | 10 | 1 | 0 |
| Notebook <br> No. 2 | 6 | 3 | 4 | 8 | 3 | 1 | 2 | 2 | 0 |

## 5. Study set T5

The total number of geometric problems in the textbook is 54 . They are of all kinds. In notebook No 1 the geometric problems are 14, in the study notebook No. 2 they are 30 and in the study notebook No. 3 the problems are 18. The total number of problems in the set is 116 . It is noteworthy the lack of balance in the number of geometric problems in the three notebooks. The number of geometric problems of two of the types is minimal: measure the length of depicted objects (6) and text problems with geometric content (9).

Table 5. Geometric content in T5

| Types | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| T5 | 7 | 12 | 4 | 9 | 5 | 1 | 4 | 10 | 2 |
| Notebook <br> No. 1 | 0 | 5 | 1 | 0 | 3 | 0 | 3 | 2 | 0 |
| Notebook <br> No. 2 | 0 | 3 | 11 | 5 | 4 | 1 | 4 | 2 | 0 |
| Notebook <br> No. 3 | 0 | 2 | 9 | 1 | 1 | 0 | 4 | 1 | 0 |

## 6. Study set T6

The total number of geometric problems in the textbook is 32 . This is the smallest number of geometric problems included in a textbook for first grade. They are of 8 species. In the notebook No. 1 , the geometric problems are 10 and in the notebook No. 2 they are 12. The total number of problems in the set is 54 . It can be said that in terms of geometric learning content this textbook has the worst performance. Of some of the species, as shown in Table 6, the problems are a total of 2,3 or 4 , which is insufficient to form students' skills.

Table 6. Geometric content in T6

| Types | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| T6 | 11 | 2 | 2 | 8 | 3 | 0 | 3 | 2 | 1 |
| Notebook <br> No. 1 | 1 | 1 | 2 | 2 | 0 | 1 | 2 | 1 | 0 |
| Notebook <br> No. 2 | 3 | 2 | 0 | 3 | 1 | 0 | 2 | 0 | 1 |

## 7. Study set T7

The total number of geometric problems in the textbook is 51 . They are of 8 types. In notebook No 1 the geometric problems are 8 , in the notebook No. 2 they are 13 and in the notebook No. 3 the problems are 25 . The total number of problems in the set is 97 . The imbalance between the geometric problems in the three study books is impressive. The number of geometric problems in two of the types is minimal: symmetry problems (7) and text-based geometric contents (9).

Table 7. Geometric content in T7

| Types | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| T7 | 14 | 15 | 4 | 12 | 3 | 0 | 1 | 1 | 1 |
| Notebook <br> No. 1 | 1 | 5 | 0 | 0 | 0 | 0 | 0 | 2 | 0 |
| Notebook <br> No. 2 | 2 | 2 | 2 | 2 | 3 | 0 | 0 | 2 | 0 |
| Notebook <br> No. 3 | 1 | 0 | 2 | 5 | 3 | 0 | 3 | 3 | 3 |

## 8. Study set T8

The total number of geometric problems in the textbook is 45 . They are of 7 types. In the notebook No. 1 the geometric problems are 8 , in the notebook No. 2 they are 24 and in the notebook No. 3 the problems are 22 . Total number of problems for the study set is 104 . It is noteworthy that in the study notebook No. 2 the segment measurement problems (4) dominate. In this study set there are at most zero positions, there are no geometric problems of certain types that are found in other study sets. The number of geometric problems of the types is also insufficient: measurement of lengths of depicted objects (6), symmetry problems (7), geometric geometry (8) and geometric text (9).

Table 8. Geometric content in T8

| Types | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| T8 | 6 | 8 | 10 | 8 | 7 | 3 | 0 | 3 | 0 |
| Notebook <br> No. 1 | 0 | 3 | 0 | 0 | 0 | 0 | 2 | 3 | 0 |
| Notebook <br> No. 2 | 1 | 4 | 6 | 11 | 1 | 0 | 0 | 0 | 1 |
| Notebook <br> No. 3 | 1 | 7 | 3 | 6 | 2 | 1 | 0 | 0 | 2 |

## 9. Study set T9

The total number of geometric problems in the textbook is 69 . They are of all kinds. In study book No. 1 the geometric problems are 13 , in the study notebook No. 2 they are 27 and in the study notebook No. 3 they are 11. The total number of problems in the study set is 120 . The imbalance in the number of geometric problems in the three notebooks is impressive. A serious disadvantage of the study set is the presence of only one text problem with geometric content. The number of geometric problems in study notebook No 1 and workbook No. 3 is insufficient.

| Table 9. Geometric content in T9 |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Types | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| T9 | 15 | 14 | 5 | 10 | 9 | 6 | 7 | 2 | 1 |
| Notebook <br> No. 1 | 2 | 4 | 2 | 0 | 0 | 0 | 1 | 4 | 0 |
| Notebook <br> No. 2 | 3 | 5 | 5 | 5 | 3 | 4 | 2 | 0 | 0 |
| Notebook <br> No. 3 | 2 | 4 | 3 | 1 | 0 | 0 | 1 | 0 | 0 |

## 10. Comparison between analyzed study sets

In analyzing the geometric problems and exercises in the nine study sets approved by the Ministry of Education and Science and the teachers, the following was found: T1 contains 149 geometric problems and occupies the first place. Next is T4 with 134 problems. At third place, T2 and T9 with 120 problems. This is also approximately the number of geometric problems in T5 - 116 problems. Three of the study sets have comparable indicators for the number of geometric problems: T8 (104), T7 (97) and T3 (96). The last ranked is the study set T6 with a minimum number of geometric contents -54 problems.

Table 10. Total number of geometric problems and exercises in study sets

| Study sets | T1 | T2 | T3 | T4 | T5 | T6 | T7 | T8 | T9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Geometric <br> problems <br> (total <br> number) | 149 | 120 | 96 | 134 | 116 | 54 | 97 | 104 | 120 |

## 3. RESULTS

The comparative study of the approved first grade mathematics sets on the geometric problems and exercises contained therein, the following conclusions can be drawn:

1. The relative share of geometric problems and exercises was increased as compared with the up to now study kits in mathematics. This is proof that the authors of these textbooks are aware of the importance of geometric learning content for achieving the objectives of mathematics study.
2. The total number of geometric problems and exercises in different study sets varies greatly. From 149 in T 1 to 54 in T6. This ratio is approximately 3 to 1 . This makes the different mathematics study sets unequal. It is important that when teachers choose a study set to work on, they should take this fact into account.
3. The analysis show that there are types of geometric problems of which individual authors' collectives either did not include any problem or included 1 to 4 problems in total in the textbooks and notebooks. By solving one geometric problem, it is not possible to build any learning skills in the first grade.
4. A major weakness of most study sets is the absence or minimum number of text problems with geometric content. This would negatively affect the geometry study in the next elementary school grades.
5. When measuring the learning outcomes of mathematics at the end of the fourth grade (the initial stage), difficulties are found for Bulgarian students in solving geometric problems. This calls for new geometric content to be included in the new study sets. In textbooks, and especially in notebooks, there should be a significant number of geometric problems and exercises from the various species. Such are the study sets T1, T5 and T7.

## 4. CONCLUSION

The comparative study of mathematical content in new mathematics study sets for the first grade shows that there are a number of significant differences in the volume and type of variety of problems and exercises with geometric content. In some of the analyzed sets, there is a lack of a systematic methodical approach when integrating this element of the curriculum. The conclusions reached are essential for the authors of mathematics study sets for the second, third and fourth grades of the Bulgarian primary school. These conclusions can guide primary teachers when choosing a study set to teach in the first grade.

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