A STOCHASTIC MORATLITY MODEL FOR THE ALBANIAN LIFE INSURANCE COMPANIES

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Abstract: Due to increased focus on risk management and measurement for insurers, the work with mortality models has developed in Albania in the last years. As source of data, we have used the Mortality Tables of Census of Population in Albania compiled by INSTAT (The National Institute of Statistics of Albania) this last decennium.
Two well-known mortality stochastic models are used on these data. The results of these studied models, represented by sets of Albanian coefficients, are compared with values of the mortality tables, used a priori by our life insurance companies (existing model), aiming to create an applicable model for the whole age ranges.
However, at a certain age range, there are disadvantages to these models. The goal of this paper is to clarify which model is more suitable for a certain age range, taking into consideration the mortality rate of our country.
We use the improved one factor of Lee and Carter model, which treats the future mortality index as a simple Random Walk with Drift. The results of this model show that the Albanian coefficients of mortality are lower than those of existing models at all age-groups, except the highest. It reflects the fact that mortality was, on average, lower in Albania from 2006 to 2018 than in our life insurance models. These mortality data estimated by this method, can be used as the basis for forecasting. But the old age-specific rate is so low that they cannot be projected to decline much further. (Coefficient describes variation of the death rate at old age, as well as the relative sensitivity of death rates to variation, when the general level of mortality changes.)
The other model used is the stochastic mortality model of Cairns et. al (2007). The central form of this mortality model is more suitable to capture the cohort effect in the data. We come to the conclusion, that the second model suitable for the old age-groups.
Keywords: age-specific parameter, mortality rate, an appropriate model, estimation.

1. INTRODUCTION

All kinds of life insurance policies are based on some conditions such as lifestyle and medical history. The life insurance industry relies heavily on mortality tables.
A mortality table shows the rate of deaths occurring in a defined population during a selected time interval. Statistics included in a mortality table show the probability of a person's death based on their age, or the number of people per 1,000 living who are expected to die before their next birthday. Our life insurance companies use mortality tables of neighboring countries, such as Croatia, to help determine premiums and to make sure the insurance company remains solvent.
This proceeding way was found to be useful, as the historical data were not large enough to make appropriate projections. So, they wouldn't represent the insurance portfolio of a life insurer appropriately.
Due to the increasing focus of risk management and measurement for insurer, the work with mortality models has developed in the last years in our country.

2. STOCHASTIC MORTALITY MODELS

In this paper we will try to bring an updated mortality rate to the table, which may service the Albanian life insurers, using the most common statistical and actuarial methods.
The central mortality rate is

$$m_{x,t} = \frac{D_{x,t}}{E_{x,t}} = \frac{\#\,death\ during\ calendar\ year\ t,\ aged\ x\ last\ birthday}{Estimated\ mid-year\ population\ aged\ x\ that\ year}$$

First of all, the data matrix in log mortality rates for the male and female population from 2006 to 2018 is taken. In particular, the rows of the matrix represent the 21 age groups [0], [1–4], [5–9] ..., [95–99] and the columns refer to the years 2006–2018.
Below, the one factor model of Lee and Carter is used
$\ln(m_{x,t}) = \alpha_x + b_x K_t + \epsilon_{x,t}$ (1)
This equation describes the logarithm of a time series of age-central mortality rates $m_{x,t}$ as the sum of :

- α_x - an age-specific parameter that is independent of time

- a component given by the product of a time-varying parameter K_t with b_x- which represents the trend of mortality at each age-group (how quickly or slowly it's generated when the general level of mortality rate changes)
- $\epsilon_{x,t}$ - the error term, assumed to be with mean 0 , variance σ2

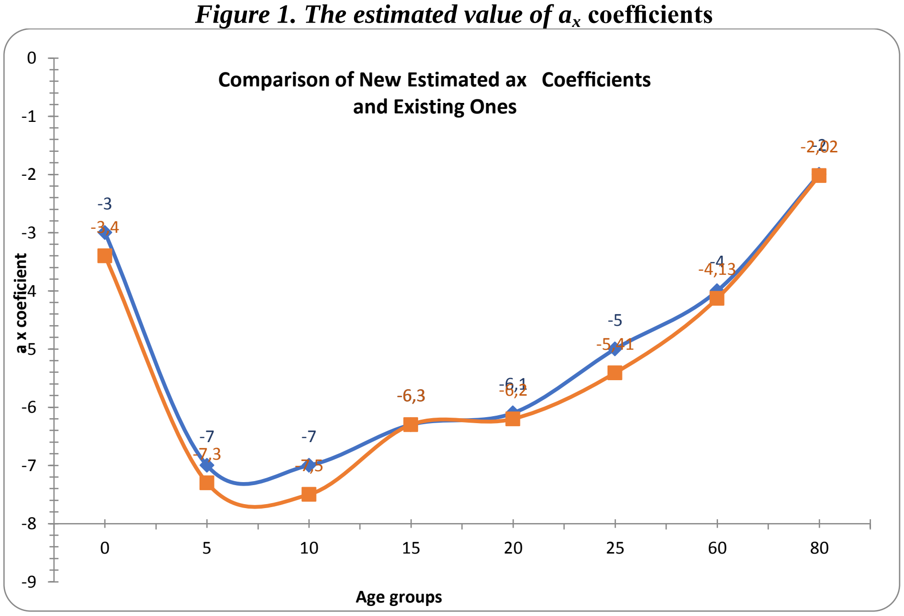
Inspection of the first equation leads to the following notes, the right-hand side of the equation includes no observed variable, so ordinary regression methods cannot be used to estimate the model.
If a_x, b_x, and K_t form one set of coefficients for the model, then a_x, b_x/A, and $A \times k_t$ will be an exactly equivalent set for any constant A.
Similarly, $a_x - b_x \times A$, b_x, k_t (1 + A) will be an equivalent formulation for arbitrary constant A.
Lee-Carter suggested a unique representation (the Singular Value Decomposition, SVD) by setting:

- a_x equal to the average of the logarithms of $m_{x,t}$ over the data period,
- the average value of k_t equal to zero
- In this case the sum of the b_x values is unity.

The values of k_t form a time series, with one value for each year of data. Standard statistical methods then can be used to model and forecast this time series. Instead we assumed that the "random walk with drift" model held, based on Lee -Carter improvement, that selected a random walk with drift as the appropriate model.
$k_t = k_{t-1} + c + e_t$. (2)
c is the drift term

- k_t is forecast to decline linearly with increments of c
- e_t , deviations from this path are incorporated permanently in the trajectory.
  We used it to forecast k_t over the desired time range.

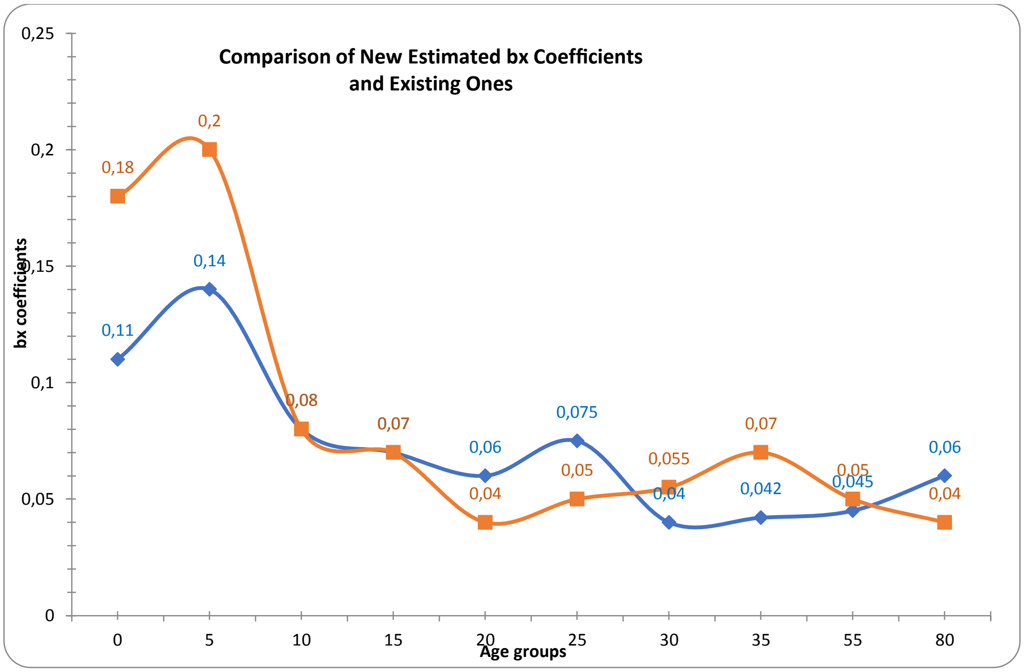
***Figure 1. The estimated value of a_x* coefficients**



Note: New Albanian results Existing results

As we know, a_x are just the average values of the logarithms of death rates.
The new Albanian coefficients lie under those of existing model, at all age-groups except the highest, reflecting the fact that mortality was, on average, lower in Albania from 2006 to 2018 than estimated in the existing model.

***Figure 2. The estimated value of b_x* coefficients**



Note: New Albanian results Existing results

The b_x coefficients describe variation of the death rate at age x, when the general level of mortality changes, as well as the relative sensitivity of death rates to variation in the k_t parameter.

- We note that sets of new coefficients for the Albania and old ones look quite similar.
- With 21 age groups, if $b_x= 1/21$ for all x, then all the rates would move up and down proportionately, maintaining constant ratios to one another.

However, it can be seen, in fact, some ages are more sensitive than others.

➢ b_x reaches high values for some x, (i.e. x=0 , in which case we can say that the death rate for infant mortality varies greatly when the general level of mortality changes)

➢ b_x is small than the death rate at such ages varies little with changes in the general level of mortality (as the cases with mortality at older ages).

Generally speaking, the younger the age, the greater its sensitivity to variation in the k_t parameter. It is noticed that the residuals in the age groups 1–4, 5–9, 15–19 and 35–39 are far from being homogeneous.

Finally:

➢ This method can be used as the basis for a simple-model life table system. This model allows the possibility to describe everything accurately, even as the age pattern of mortality changes over time.

➢ The a_x coefficients will always change, over different historical periods, because they are the average logarithm of death rates.

➢ Some argue that many age-specific rates are so low that they cannot be projected to decline much further. So, as shown in our mortality table, age-specific rates are low, at old ages.

3. A STOCHASTIC MORTALITY MODEL OF CAIRNS et. al (2007)

It is known that the better model for old ages is the stochastic mortality model of Cairns. He observed that, for data from England and the US, the fitted cohort effect appeared to have a trend in the year of birth. To capture the cohort effect in the data, the central mortality model shown below, is proposed:

$$\log m(x, t) = \alpha x + \kappa_{(1)t} + \kappa_{(2)t}(x - \bar{x}) + \gamma_{t-x} \quad (3)$$

where $\bar{x}$ is the mean age in sample range ($\kappa_{(1)t}$, $\kappa_{(2)t}$) which is assumed to be a bivariate random walk with drift.

- The ax is similar as in the Lee-Carter model and makes sure that the basic shape of mortality is in line with historical observations.
- The factor $\kappa_{(1)t}$ is a random period effect that represents changes in the level of mortality for all ages.

- The factor $\kappa_{(2)t}$ allows changes in mortality to vary between ages and to reflect the historical observation so that improvement rates can vary for different age groups.
- The factor γ_{t-x} is a random cohort effect expressed as a function of the year of birth (t-x).

Next to ax, the model has 3 stochastic factors ($\kappa_{(1)t}$ $\kappa_{(2)t}$ γ_{t-x})
This model has similar identifiability problems to the previous model.
Therefore some restrictions are imposed

$$\sum \gamma_{t-x} = 0$$

where sum is from the earliest to latest year of birth to which a cohort effect is fitted.
We have found this model more suitable for old ages.

4. CONCLUSIONS

All well known stochastic mortality models have nice features but also disadvantages. We have proposed a stochastic mortality model that takes in consideration the mortality rate of our country.
Due to a (Lee- Carter type) variable, that describes the shape of the mortality curve over ages, and as conclusion of a separate stochastic factor for old ages, the model is applicable to a full age range.

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