

MTPL CLAIMS UNCERTAINTY IN THE CHAIN LADDER METHOD

Kleida Haxhi

Department of Mathematical Engineering, Faculty of Mathematics and Physics Engineering,
Polytechnic University of Tirana, Albania, k.haxhi@fimif.edu.al

Abstract: Technical reserves, especially claims reserves are an important issue in a non-life insurance company. Under Albanian law reporting is done every quarter as well as the company's financial statements. The value of technical reserves affects directly the company's technical result.

There are several methods for estimations the technical claims reserves. Initially, most of these methods began as deterministic algorithms. Over time actuaries began developing and analyzing stochastic models that justify these algorithms. These stochastic models enable analysis and quantification of the uncertainty of forecasting responsibilities for outstanding claims. Some of the models used are: The Poisson model, the over-dispersed Poisson model, Gamma model, Negative binomial model, and the Log-normal model. Parametric models such Wright's model and Bootstrap are also used. General linear models constitute a flexible class of stochastic models and are available in the analysis of future payments.

Chain ladder model developed by Mack is the more prevalent model. This model is based on the triangle of development of incurred or paid claims and it is free distribution and also it does not require additional information. Based on the model of Mack, there are also developed other models easily applicable. Different methods yield different results, often similar to each other, but also different between them. These results are influenced by the available data. From the application made, it reached the conclusion that the data are often uncertain.

The technical claims reserves, as all technical reserves directly affecting profit loss statement, as well as the technical balance of the company, it is required as fair evaluation of them. Results of application of stochastic methods are highly dependent on the reliability and accuracy of data. The actuary seeing the progress and history of claims in a portfolio, the market where are developed claims payments over the years, the values of outstanding claims, claims in process court, which values estimates is more appropriate to establish technical reserves. Also the insurance company must hold sufficient assets to cover technical reserves. The value of assets covering technical provisions must at all times be not less than the gross amount of technical reserves.

Stochastic methods of reserves estimation discussed in this paper serve to assess the technical provisions of outstanding claims, as well as forecast cash payment of claims in the coming years.

Keywords: stochastic methods, chain ladder model, uncertainty of data.

1. CHAIN LADDER METHOD

The chain-ladder technique uses cumulative data, and derives a set of 'development factors' or 'link ratios'. To a large extent, it is irrelevant whether incremental or cumulative data are used when considering claims reserving in a stochastic context, and it is easier for the explanations here to use incremental. In order to keep the exposition as straightforward as possible, and without loss of generality, we assume that the data consist of a triangle of incremental claims. This is the simplest shape of data that can be obtained, and it is often the case that data from early origin years are considered fully run-off or that other parts of the triangle are missing. Using a triangle avoids us having to introduce complicated notation to confront with all possible situations. Thus, we assume that we have the following set of incremental claims data: $C_{ij}; i=1, \dots, n; j=1, \dots, n-i+1$

The suffix i refers to the row, and could indicate accident year or underwriting year. The suffix j refers to the column, and indicates the delay, assumed also to be measured in years or quarterlies. The cumulative claims are defined by:

$$D_{ij} = \sum_{k=1}^j C_{ik}$$

The chain-ladder technique estimates the development factors as:

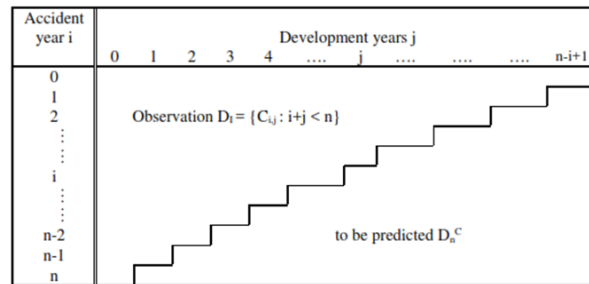
$$\hat{f}_j = \frac{\sum_{i=1}^{n-j+1} D_{ij}}{\sum_{i=1}^{n-j+1} D_{i,j-1}}$$

These are then applied to the latest cumulative claims in each row $D_{i,n-i+1}$ to produce forecasts of future values of cumulative claims:

$$\hat{D}_{i,n-i+2} = D_{i,n-i+1} \hat{f}_{n-i+2}$$

$$\hat{D}_{i,k} = \hat{D}_{i,k-1} \hat{f}_k \quad k=n-i+3, n-i+4, \dots, n$$

The chain-ladder technique, in its simplest form, consists of a way of obtaining forecasts of ultimate claims only. Here 'ultimate' is interpreted as the latest delay year so far observed, and does not include any tail factors. From a statistical viewpoint, given a point estimate, the natural next step is to develop estimates of the likely variability in the outcome so that assessments can be made, for example, of whether extra reserves should be held for prudence, over and above the predicted values. In this respect, the measure of variability commonly used is the prediction error, defined as the standard deviation of the distribution of possible reserve outcomes. It is also desirable to take account of other factors, such as the possibility of unforeseen events occurring which might increase the uncertainty, but which are difficult to model. The first step to obtaining the prediction error is to formulate an underlying statistical model making assumptions about the data. If the aim is to provide a stochastic model which is analogous to the chain-ladder technique, then an obvious first requirement is that the predicted values should be the same as those of the chain-ladder technique. There are two ways in which this has been attempted: specifying distributions for the data; or just specifying the first two moments.



2. VARIABILITY OF THE CHAIN LADDER METHOD

C_{ik} denote the accumulated total claims amount of accident year i , $1 \leq i \leq n$, paid or incurred up to development year k , $1 \leq k \leq n$. The values of C_{ik} for $i+k \leq n+1$ are known and we want to estimate the values of C_{ik} for $i+k > n+1$, in particular the ultimate claims amount C_{in} for each accident year $i=2, \dots, n$. Then

$$R_i = C_{in} - C_{i,n+1-i}$$

is the outstanding claims reserve of accident year i , as $C_{i,n+1-i}$ has already been paid or incurred up to now.

The chain ladder method consist of estimating the ultimate claims by

$$C_{in} = C_{i,n+1-i} \cdot f_{n+1-i} \cdot \dots \cdot f_{n-1} \quad 2 \leq i \leq n \quad (1)$$

where

$$f_k = \frac{\sum_{j=1}^{n-k} C_{j,k+1}}{\sum_{j=1}^{n-k} C_{j,k}} \quad 1 \leq k \leq n-1 \quad (2)$$

are called age-to-age factors.

This manner of projecting the known claims amount $C_{i,n+1-i}$ to the ultimate claims amount C_{in} uses for all accident yeras $i \geq n+1-k$ the same factor f_k for the increase of the claims amount from the development year k to $k+1$, although the observed individual development factors $C_{i,k+1}/C_{ik}$ of the accident year $i \leq n-k$ are usually different from one another and from f_k . And the end of the development year k we have consider $C_{i,k+1}$ and C_{in} as random variables whereas the realizations C_{i1}, \dots, C_{ik} are known to us and therefore no longer random variables but scalars. For the purposes of analysis every C_{ik} can be a random variable or scalar depending on the development year at the end of whether C_{ik} belongs to the known part $i+k \leq n+1$ of run-off triangle or not. When taking expected values or variances we therefore must always also state the development year at the end of which we imagine to be. The chain ladder method assumes the existence of accident year independent factors f_1, \dots, f_{n-1} such that, given the development C_{i1}, \dots, C_{ik} , the realization of $C_{i,k+1}$ is close to $C_{ik}f_k$, the latter being the expected value of $C_{i,k+1}$

$$E(C_{i,k+1} | C_{i1}, \dots, C_{ik}) = C_{ik}f_k \quad 1 \leq i \leq n; 1 \leq k \leq n-1 \quad (3)$$

This formula is a conditional expected value. These equations constitute an assumption which is not imposed by us but rather implicitly underlies the chain ladder method. This is based on two aspects of the basic chain ladder equations (1): one is the fact that (1) uses the same age to age factors f_k for different accident years $i = n+1-k, \dots, n$. Therefore equations (3) also postulate age to age parameters f_k which are the same for all accident years. The other is the fact that (1) uses only the most recent observed value $C_{i,n+1-i}$ as basis for the projection to ultimate

ignoring on the one hand all amounts $C_{i1}, \dots, C_{i,n-1}$ observed earlier and on the other hand the fact that $C_{i,n+1-i}$ could substantially deviate from its expected value. It be also possible to project to ultimate the amounts $C_{i1}, \dots, C_{i,n-1}$ of the earlier development years with the help of age to age factors f_1, \dots, f_{n-1} , and to combine all these projected amounts together with $C_{i,n+1-i} \cdot f_{n+1-i} \cdot \dots \cdot f_{n-1}$ into a common estimator C_{in} . It would also be possible to use the values $C_{j,n+1-i}$ of earlier accident years $j < i$ as additional estimators for $E(C_{i,n+1-i})$ by translating them into accident year i with help of measure of volume for each accident year. We can rewrite (3) into the form

$$E(C_{i,k+1} | C_{i1}, \dots, C_{ik}) = f_k$$

because C_{ik} is a scalar under the condition that we know C_{i1}, \dots, C_{ik} . This form of (3) shows that the expected value of the individual development factor $C_{i,k+1}/C_{ik}$ equalto f_k irrespective of the prior development $C_{ik}/C_{i,k-1}$. The subsequent development factors $C_{ik}/C_{i,k-1}$ and $C_{i,k+1}/C_{ik}$ are uncorrelated. This means that after a rather high value of $C_{ik}/C_{i,k-1}$ the expected size of the next development factors $C_{i,k+1}/C_{ik}$ is the same as after a rather low value of $C_{ik}/C_{i,k-1}$. For this reason we should not apply the chain ladder method to a business where we usually observe a rather small increase $C_{i,k+1}/C_{ik}$ if $C_{ik}/C_{i,k-1}$ is higher than in most other accident years.

2.1 Analysis of age-to-age factors

$$f_k = \frac{\sum_{j=1}^{n-k} C_{j,k+1}}{\sum_{j=1}^{n-k} C_{jk}} = \sum_{j=1}^{n-k} \frac{C_{jk}}{\sum_{j=1}^{n-k} C_{jk}} \cdot \frac{C_{j,k+1}}{C_{jk}}$$

$C_{j,k+1}/C_{jk}$, $1 \leq j \leq n - k$, is an unbiased estimator of f_k because

$$D(C_{j,k+1}/C_{jk} | C_{j1}, \dots, C_{jk}) = \alpha_k^2 / C_{jk}$$

$$D(C_{j,k+1} | C_{j1}, \dots, C_{jk}) = C_{jk} \alpha_k^2 \quad 1 \leq j \leq n, \quad 1 \leq k \leq n - 1 \quad (5)$$

with constant α_k^2 , $1 \leq k \leq n - 1$.

2.2 Measuring the variability of the ultimate claims

$$C_{in} = C_{i,n+1-i} \cdot f_{n+1-i} \cdot \dots \cdot f_{n-1}$$

$$E(\tilde{C}_{in}) = E(C_{in}), \text{ for } 2 \leq i \leq n.$$

$$mse(C_{in}) = E((C_{in} - \tilde{C}_{in})^2 | D)$$

where $D = \{C_{ik} | i + k \leq n + 1\}$ are the observed values.

$$mse(C_{in}) = D(C_{in} | D) + (E(C_{in} | D) - C_{in})^2$$

$$\tilde{R}_i = \tilde{C}_{in} - C_{i,n+1-i}$$

$$R_i = C_{in} - C_{i,n+1-i}$$

$$mse(R_i) = E((\tilde{R}_i - R_i)^2 | D) = E((\tilde{C}_{in} - C_{in})^2 | D) = mse(C_{in})$$

$$s.e.(\tilde{R}_i) = s.e.(\tilde{C}_{in})$$

$$(s.e.(\tilde{C}_{in}))^2 = \tilde{C}_{in}^2 \sum_{k=n+1-i}^{n-1} \frac{\tilde{\alpha}_k^2}{\tilde{f}_k^2} \left(\frac{1}{\tilde{C}_{ik}} + \frac{1}{\sum_{j=1}^{n-k} C_{jk}} \right) \quad (6)$$

$$\tilde{\alpha}_k^2 = \frac{1}{n-k-1} \sum_{j=1}^{n-k} C_{jk} \left(\frac{C_{j,k+1}}{C_{jk}} - \tilde{f}_k \right)^2 \quad 1 \leq k \leq n-2 \quad (7)$$

$$\tilde{C}_{ik} = C_{i,n+1-i} \cdot \tilde{f}_{n+1-i} \cdot \dots \cdot \tilde{f}_{k-1}, \quad k > n+1-i$$

3. UNCERTAINTY IN THE METHOD

The most important portfolio of the general insurance in Albania is Motor Third Party Liability, MTPL. It constitutes more than 65% of all premiums. Hence the reserves estimation for claims deriving from these policies is the main issue for non-life actuaries. The data used in this paper are from one of the non-life company in the Albanian market. For the triangles we consider the quarterly data, domestic third party liability paid and incurred claims from 2014 to 2019. Based on the incurred and paid calculation, the results, triangle reserve, standard error amounts and percentage standard errors are presented in the table below:

| Accident Quarter | Incurred | | | Paid | | | |
|------------------|--------------------|-----------------------|------------------|------------------|--------------------|-----------------------|------------------|
| | Triangle Reserve | Standard Error Amount | Standard Error % | Accident Quarter | Triangle Reserve | Standard Error Amount | Standard Error % |
| 2014 Q1 | 0 | 0 | 0.00% | 2014 Q1 | 0 | 0 | 0.00% |
| 2014 Q2 | 2,905 | 4,613 | 158.78% | 2014 Q2 | 15,513 | 85,392 | 550.47% |
| 2014 Q3 | 4,579 | 5,785 | 126.35% | 2014 Q3 | 34,427 | 84,227 | 244.65% |
| 2014 Q4 | 165,546 | 362,767 | 219.13% | 2014 Q4 | 59,716 | 140,550 | 235.36% |
| 2015 Q1 | 402,805 | 601,625 | 149.36% | 2015 Q1 | 189,474 | 241,720 | 127.57% |
| 2015 Q2 | 438,479 | 214,116 | 48.83% | 2015 Q2 | 141,342 | 148,622 | 105.15% |
| 2015 Q3 | 1,322,580 | 2,045,104 | 154.63% | 2015 Q3 | 307,721 | 907,633 | 294.95% |
| 2015 Q4 | 3,164,828 | 4,384,856 | 138.55% | 2015 Q4 | 579,175 | 1,113,485 | 192.25% |
| 2016 Q1 | 2,258,760 | 3,822,733 | 169.24% | 2016 Q1 | 377,760 | 813,211 | 215.27% |
| 2016 Q2 | 3,655,975 | 4,904,925 | 134.16% | 2016 Q2 | 498,341 | 922,421 | 185.10% |
| 2016 Q3 | 5,854,392 | 4,135,893 | 70.65% | 2016 Q3 | 864,313 | 1,151,162 | 133.19% |
| 2016 Q4 | 3,619,212 | 1,736,570 | 47.98% | 2016 Q4 | 1,467,909 | 1,603,577 | 109.24% |
| 2017 Q1 | 9,082,175 | 8,514,913 | 93.75% | 2017 Q1 | 1,367,399 | 1,417,111 | 103.64% |
| 2017 Q2 | 9,008,542 | 8,227,772 | 91.33% | 2017 Q2 | 1,858,305 | 1,763,219 | 94.88% |
| 2017 Q3 | 7,385,068 | 2,064,035 | 27.95% | 2017 Q3 | 2,877,405 | 2,226,550 | 77.38% |
| 2017 Q4 | 4,402,683 | 2,135,849 | 48.51% | 2017 Q4 | 3,056,971 | 2,333,451 | 76.33% |
| 2018 Q1 | 12,303,818 | 10,613,287 | 86.26% | 2018 Q1 | 3,582,702 | 2,583,403 | 72.11% |
| 2018 Q2 | 22,275,992 | 15,261,367 | 68.51% | 2018 Q2 | 6,063,577 | 3,724,198 | 61.42% |
| 2018 Q3 | 16,710,065 | 12,184,902 | 72.92% | 2018 Q3 | 7,424,092 | 4,129,506 | 55.62% |
| 2018 Q4 | 26,018,919 | 6,383,698 | 24.53% | 2018 Q4 | 14,969,307 | 7,188,752 | 48.02% |
| 2019 Q1 | 14,797,319 | 5,232,173 | 35.36% | 2019 Q1 | 28,170,278 | 10,249,286 | 36.38% |
| 2019 Q2 | 46,798,203 | 25,899,833 | 55.34% | 2019 Q2 | 24,456,445 | 9,793,739 | 40.05% |
| 2019 Q3 | 12,147,382 | 7,204,591 | 59.31% | 2019 Q3 | 35,414,205 | 12,157,190 | 34.33% |
| 2019 Q4 | 9,659,101 | 2,582,722 | 26.74% | 2019 Q4 | 18,658,027 | 9,221,751 | 49.43% |
| Total | 211,479,329 | 128,524,129 | 60.77% | Total | 152,434,402 | 74,000,157 | 48.55% |

The three assumptions of the chain ladder method are:

- $E(C_{i,k+1} | C_{i1}, \dots, C_{ik}) = C_{ik} f_k$
- variables $\{C_{i1}, \dots, C_{in}\}$ and $\{C_{j1}, \dots, C_{jn}\}$ of different accident year $i \neq j$ are independent
- $D(C_{j,k+1} | C_{j1}, \dots, C_{jk}) = C_{jk} \alpha_k^2$

$$\sum_{i=1}^{n-k} (C_{i,n+1} - C_{ik} f_k)^2 = \text{minimum}$$

Cumulative paid triangle, quarterlies from 2014 to 2019

| Accident Quarter | 1 (3m) | 2 (6m) | 3 (9m) | 4 (12m) | 5 (15m) | 6 (18m) | 7 (21m) | 8 (24m) | 9 (27m) | 10 (30m) | 11 (33m) | 12 (36m) | 13 (39m) | 14 (42m) | 15 (45m) | 16 (48m) | 17 (51m) | 18 (54m) | 19 (57m) | 20 (60m) | 21 (63m) | 22 (66m) | 23 (69m) | 24 (72m) |
|------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 2014 Q1 | 3.316900 | 6.508783 | 6.557783 | 7.885026 | 9.617107 | 15.857107 | 17.987107 | 18.079107 | 18.079107 | 18.079107 | 18.079107 | 18.079107 | 18.079107 | 18.079107 | 18.079107 | 18.079107 | 18.079107 | 18.079107 | 18.079107 | 18.079107 | 18.079107 | 18.079107 | 18.079107 | 18.079107 |
| 2014 Q2 | 3.214939 | 6.866628 | 7.345703 | 11.687703 | 14.487703 | 14.487703 | 23.927703 | 27.027703 | 27.027703 | 27.027703 | 27.027703 | 27.027703 | 27.027703 | 27.027703 | 27.027703 | 27.027703 | 27.027703 | 27.027703 | 27.027703 | 27.027703 | 27.027703 | 27.027703 | 27.027703 | 27.027703 |
| 2014 Q3 | 5.276015 | 9.841315 | 9.919315 | 13.288315 | 16.125315 | 19.205315 | 19.205315 | 19.205315 | 19.205315 | 19.205315 | 19.205315 | 19.205315 | 19.205315 | 19.205315 | 19.205315 | 19.205315 | 19.205315 | 19.205315 | 19.205315 | 19.205315 | 19.205315 | 19.205315 | 19.205315 | 19.205315 |
| 2014 Q4 | 6.555188 | 11.820388 | 12.967388 | 16.961733 | 19.727022 | 19.739022 | 20.570385 | 27.445385 | 27.445385 | 27.445385 | 27.445385 | 27.445385 | 27.445385 | 27.445385 | 27.445385 | 27.445385 | 27.445385 | 27.445385 | 27.445385 | 27.445385 | 27.445385 | 27.445385 | 27.445385 | 27.445385 |
| 2015 Q1 | 10.118350 | 15.553350 | 18.269750 | 23.534950 | 25.810950 | 28.644550 | 40.111851 | 43.535108 | 43.913108 | 45.721053 | 53.982971 | 54.714533 | 55.664533 | 55.664533 | 55.664533 | 55.664533 | 55.664533 | 55.664533 | 55.664533 | 55.664533 | 55.664533 | 55.664533 | 55.664533 | 55.664533 |
| 2015 Q2 | 3.575356 | 8.674585 | 9.734885 | 9.943885 | 10.987590 | 21.122864 | 21.128864 | 25.128864 | 25.187464 | 28.098512 | 28.455272 | 28.455272 | 28.455272 | 28.455272 | 28.455272 | 28.455272 | 28.455272 | 28.455272 | 28.455272 | 28.455272 | 28.455272 | 28.455272 | 28.455272 | 28.455272 |
| 2015 Q3 | 7.913952 | 15.063348 | 16.234648 | 19.444026 | 22.588993 | 29.375993 | 30.955993 | 33.177499 | 34.788899 | 37.178300 | 37.717830 | 40.562335 | 40.568614 | 40.995886 | 44.050009 | 44.100009 | 44.100009 | 44.100009 | 44.100009 | 44.100009 | 44.100009 | 44.100009 | 44.100009 | 44.100009 |
| 2015 Q4 | 7.366624 | 15.316574 | 16.217674 | 22.415195 | 34.729946 | 39.913919 | 44.354894 | 44.462894 | 53.852614 | 62.712614 | 62.925548 | 62.925548 | 63.961935 | 64.211935 | 64.716735 | 64.716735 | 64.716735 | 64.716735 | 64.716735 | 64.716735 | 64.716735 | 64.716735 | 64.716735 | 64.716735 |
| 2016 Q1 | 8.239731 | 15.953091 | 20.646001 | 22.161070 | 24.446785 | 29.089203 | 31.239203 | 32.799203 | 36.252203 | 36.622513 | 36.622513 | 36.722513 | 37.399692 | 38.721192 | 38.721192 | 38.721192 | 38.721192 | 38.721192 | 38.721192 | 38.721192 | 38.721192 | 38.721192 | 38.721192 | 38.721192 |
| 2016 Q2 | 7.067558 | 16.817546 | 18.268225 | 19.313534 | 19.807194 | 24.366640 | 30.861299 | 36.379299 | 36.379299 | 44.230299 | 44.236512 | 44.236512 | 44.236512 | 44.236512 | 44.236512 | 44.236512 | 44.236512 | 44.236512 | 44.236512 | 44.236512 | 44.236512 | 44.236512 | 44.236512 | 44.236512 |
| 2016 Q3 | 8.622936 | 16.049991 | 16.789890 | 20.835200 | 22.034175 | 47.339275 | 50.381051 | 51.224818 | 53.471097 | 53.701097 | 53.707361 | 53.707361 | 53.707361 | 53.707361 | 53.707361 | 53.707361 | 53.707361 | 53.707361 | 53.707361 | 53.707361 | 53.707361 | 53.707361 | 53.707361 | 53.707361 |
| 2016 Q4 | 6.511123 | 15.227877 | 20.351627 | 22.884077 | 28.706186 | 33.336788 | 40.529865 | 41.266197 | 50.552901 | 50.559119 | 50.559119 | 53.274279 | 53.735823 | 53.735823 | 53.735823 | 53.735823 | 53.735823 | 53.735823 | 53.735823 | 53.735823 | 53.735823 | 53.735823 | 53.735823 | 53.735823 |
| 2017 Q1 | 5.715990 | 9.474246 | 16.406478 | 18.982458 | 19.014128 | 23.772344 | 24.465143 | 24.471302 | 24.471302 | 25.192302 | 25.192302 | 25.192302 | 25.192302 | 25.192302 | 25.192302 | 25.192302 | 25.192302 | 25.192302 | 25.192302 | 25.192302 | 25.192302 | 25.192302 | 25.192302 | 25.192302 |
| 2017 Q2 | 7.303150 | 16.015984 | 18.750255 | 21.031300 | 26.856456 | 28.260993 | 32.413452 | 34.715452 | 39.178252 | 39.207769 | 39.207769 | 39.207769 | 39.207769 | 39.207769 | 39.207769 | 39.207769 | 39.207769 | 39.207769 | 39.207769 | 39.207769 | 39.207769 | 39.207769 | 39.207769 | 39.207769 |
| 2017 Q3 | 7.903088 | 16.012112 | 21.746355 | 30.993085 | 34.397374 | 51.249398 | 55.340741 | 55.340741 | 57.201094 | 58.830356 | 58.830356 | 58.830356 | 58.830356 | 58.830356 | 58.830356 | 58.830356 | 58.830356 | 58.830356 | 58.830356 | 58.830356 | 58.830356 | 58.830356 | 58.830356 | 58.830356 |
| 2017 Q4 | 4.240097 | 14.177494 | 16.981732 | 26.234267 | 38.453187 | 38.503187 | 41.612087 | 41.612087 | 41.612087 | 41.612087 | 41.612087 | 41.612087 | 41.612087 | 41.612087 | 41.612087 | 41.612087 | 41.612087 | 41.612087 | 41.612087 | 41.612087 | 41.612087 | 41.612087 | 41.612087 | 41.612087 |
| 2018 Q1 | 7.021044 | 14.281747 | 17.282932 | 26.274214 | 27.069974 | 28.189193 | 32.021205 | 33.310869 | 33.310869 | 33.310869 | 33.310869 | 33.310869 | 33.310869 | 33.310869 | 33.310869 | 33.310869 | 33.310869 | 33.310869 | 33.310869 | 33.310869 | 33.310869 | 33.310869 | 33.310869 | 33.310869 |
| 2018 Q2 | 7.164444 | 18.746601 | 21.039520 | 25.044520 | 28.230640 | 37.951635 | 37.998410 | 37.998410 | 37.998410 | 37.998410 | 37.998410 | 37.998410 | 37.998410 | 37.998410 | 37.998410 | 37.998410 | 37.998410 | 37.998410 | 37.998410 | 37.998410 | 37.998410 | 37.998410 | 37.998410 | 37.998410 |
| 2018 Q3 | 5.543291 | 16.072527 | 17.883153 | 19.864498 | 21.000863 | 28.180214 | 28.180214 | 28.180214 | 28.180214 | 28.180214 | 28.180214 | 28.180214 | 28.180214 | 28.180214 | 28.180214 | 28.180214 | 28.180214 | 28.180214 | 28.180214 | 28.180214 | 28.180214 | 28.180214 | 28.180214 | 28.180214 |
| 2018 Q4 | 5.742551 | 16.159783 | 19.843416 | 26.190549 | 30.308140 | 30.308140 | 30.308140 | 30.308140 | 30.308140 | 30.308140 | 30.308140 | 30.308140 | 30.308140 | 30.308140 | 30.308140 | 30.308140 | 30.308140 | 30.308140 | 30.308140 | 30.308140 | 30.308140 | 30.308140 | 30.308140 | 30.308140 |
| 2019 Q1 | 5.520738 | 16.191982 | 26.428570 | 38.597569 | 38.597569 | 38.597569 | 38.597569 | 38.597569 | 38.597569 | 38.597569 | 38.597569 | 38.597569 | 38.597569 | 38.597569 | 38.597569 | 38.597569 | 38.597569 | 38.597569 | 38.597569 | 38.597569 | 38.597569 | 38.597569 | 38.597569 | 38.597569 |
| 2019 Q2 | 5.368037 | 14.303001 | 20.499629 | 20.499629 | 20.499629 | 20.499629 | 20.499629 | 20.499629 | 20.499629 | 20.499629 | 20.499629 | 20.499629 | 20.499629 | 20.499629 | 20.499629 | 20.499629 | 20.499629 | 20.499629 | 20.499629 | 20.499629 | 20.499629 | 20.499629 | 20.499629 | 20.499629 |
| 2019 Q3 | 8.669164 | 22.168637 | 22.168637 | 22.168637 | 22.168637 | 22.168637 | 22.168637 | 22.168637 | 22.168637 | 22.168637 | 22.168637 | 22.168637 | 22.168637 | 22.168637 | 22.168637 | 22.168637 | 22.168637 | 22.168637 | 22.168637 | 22.168637 | 22.168637 | 22.168637 | 22.168637 | 22.168637 |
| 2019 Q4 | 4.004980 | 4.004980 | 4.004980 | 4.004980 | 4.004980 | 4.004980 | 4.004980 | 4.004980 | 4.004980 | 4.004980 | 4.004980 | 4.004980 | 4.004980 | 4.004980 | 4.004980 | 4.004980 | 4.004980 | 4.004980 | 4.004980 | 4.004980 | 4.004980 | 4.004980 | 4.004980 | 4.004980 |

The solution is:

$$f_{k0} = \sum_{i=1}^{n-k} C_{ik} C_{i,k+1} / \sum_{i=1}^{n-k} C_{ik}^2$$

Development factors from 2014 to 2019

| Development Factors | 1 (3m) | 2 (6m) | 3 (9m) | 4 (12m) | 5 (15m) | 6 (18m) | 7 (21m) | 8 (24m) | 9 (27m) | 10 (30m) | 11 (33m) | 12 (36m) | 13 (39m) | 14 (42m) | 15 (45m) | 16 (48m) | 17 (51m) | 18 (54m) | 19 (57m) | 20 (60m) | 21 (63m) | 22 (66m) | 23 (69m) | 24 (72m) |
|---------------------|--------|--------|--------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 2014 Q1 | 1.96 | 1.01 | 1.16 | 1.27 | 1.62 | 1.15 | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.08 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.01 | 1.00 | 1.00 |
| 2014 Q2 | 2.14 | 1.07 | 1.59 | 1.24 | 1.00 | 1.65 | 1.13 | 1.03 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 2014 Q3 | 1.87 | 1.01 | 1.34 | 1.21 | 1.19 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.05 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 2014 Q4 | 1.80 | 1.10 | 1.31 | 1.16 | 1.00 | 1.04 | 1.33 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 2015 Q1 | 1.54 | 1.17 | 1.29 | 1.10 | 1.11 | 1.40 | 1.09 | 1.01 | 1.04 | 1.18 | 1.01 | 1.02 | 1.00 | 1.00 | 1.04 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 2015 Q2 | 2.43 | 1.12 | 1.02 | 1.10 | 1.92 | 1.00 | 1.19 | 1.00 | 1.12 | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.05 | 1.00 | 1.00 | 1.00 | 1.00 |
| 2015 Q3 | 1.90 | 1.08 | 1.20 | 1.16 | 1.30 | 1.05 | 1.07 | 1.05 | 1.08 | 1.00 | 1.08 | 1.00 | 1.01 | 1.07 | 1.00 | 1.00 | 1.00 | 1.06 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 2015 Q4 | 2.08 | 1.06 | 1.38 | 1.55 | 1.15 | 1.11 | 1.00 | 1.21 | 1.16 | 1.00 | 1.00 | 1.02 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 2016 Q1 | 1.94 | 1.29 | 1.07 | 1.10 | 1.19 | 1.07 | 1.05 | 1.11 | 1.01 | 1.00 | 1.00 | 1.00 | 1.02 | 1.04 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 2016 Q2 | 2.38 | 1.09 | 1.06 | 1.03 | 1.23 | 1.27 | 1.18 | 1.00 | 1.22 | 1.00 | 1.00 | 1.00 | 1.00 | 1.07 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 2016 Q3 | 1.86 | 1.05 | 1.24 | 1.06 | 2.15 | 1.06 | 1.02 | 1.04 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 2016 Q4 | 2.34 | 1.34 | 1.12 | 1.25 | 1.18 | 1.20 | 1.02 | 1.23 | 1.00 | 1.00 | 1.05 | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 2017 Q1 | 1.66 | 1.73 | 1.16 | 1.00 | 1.25 | 1.03 | 1.00 | 1.00 | 1.03 | 1.00 | 1.63 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 2017 Q2 | 2.19 | 1.17 | 1.12 | 1.28 | 1 | | | | | | | | | | | | | | | | | | | |

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