
STATE SPACE MODEL OF THE MECHANICAL SUBSYSTEM OF THE ELEVATOR MECHATRONIC SYSTEMS

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Abstract: One of the typical examples of mechatronic systems are modern elevators. At the same time, elevators are complex mechatronic systems that must ensure a high level of precision in operation, as well as comfort and safety for users. An integral part of the elevator mechatronic system (EMS) is the mechanical subsystem, which, like any mechanical structure, is characterized by the existence of resonant frequencies. In order for the EMS to achieve high performance, the control signals must have a rich spectrum that also contains resonant frequencies. In order to determine the resonant frequencies, we can use computer simulations. For computer simulations, a mathematical model of the mechanical subsystem of the EMS is necessary. This paper presents the derived dynamic model of the mechanical subsystem of the EMS which is in the form of state space for its easier dynamic analysis by computer simulations. The mechanical scheme which is considered in this paper consists of an elevator car and a counterweight as two lumped masses and two overhead sheaves and one drive sheave as three inertia masses. Masses are coupled by the steel wire rope. The dynamic model of wire ropes can be expressed in several ways. The rope model as an inelastic rigid body may be satisfactory in some applications but it is not suitable for derivation of a dynamic model. Hook's ideal elastic body and Standard model are often used in modeling, but in the case described in this paper they are not adequate. Hook's ideal elastic body has no modeled damping, and the Standard Model is too complex to parameterize. For derivation of the dynamic model of the mechanical subsystem of the EMS, a steel wire rope model is presented with a spring of great stiffness and with damping. Actually, Calvin's model is used and proved to be completely satisfactory. A 5 DOF model, i.e. five differential equations, was obtained, which, based on Cauchy's principle and by introducing variable states of the system, were adapted to be presented in the matrix form. Derived model it has been tested and verified through computational simulations and compared with experimental results obtained on small-scale real model of the EMS. The existence of resonant vibrations in the EMS is proved and presented. The step change of the drive torque is used for time response of the speed. Results clearly shows that the speed response has a damped oscillation. Frequencies of oscillations are equal to resonant frequency of the EMS mechanical subsystem. For frequency analysis Bode plots are used, providing much more information than time response plots. Results shows that the EMS has one resonant frequency that depends only on the structure of the EMS and does not depend on the load in the EMS's cabin. In order to be able to determine the resonant frequency or the frequency range in which the resonant frequency is located, it is necessary to use the presented dynamic model of the EMS mechanical subsystem. It is a simple and very quick tool and a mandatory first step to synthesize the speed and position control of the EMS.

Keywords: elevator, vibrations, mathematical model, state space model

1. INTRODUCTION

In 2012, around 4.3 million elevators were installed in the European Union, and 125,000 were newly installed in that year (De Almeida, 2012). The mentioned numbers clearly speak about the role and importance of elevators in modern society.

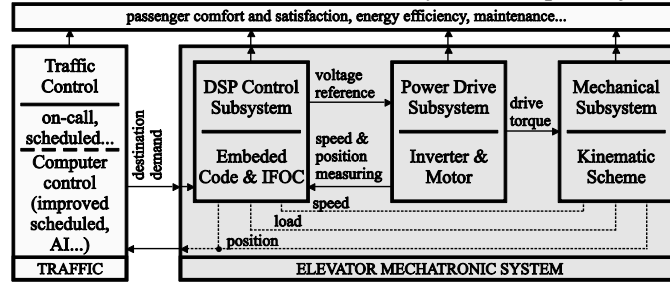
In this paper, we deal with the elevator as a mechatronic system. Its mechanical subsystem consists basically of two masses at the ends of a steel wire rope of finite stiffness (Miura, 2022). Each mechanical system and therefore the mechanical subsystem of the elevator is characterized by a resonant frequency. The resonant frequency is often within the bandwidth of the speed controller (Crespo, 2018). Therefore, the mechanical subsystem of the EMS behaves like a resonant circuit. Any sudden change in the output of the speed controller acts as an excitation to that resonant circuit. In such an excited resonant circuit, vibrations occur and the function of the controller is blocked.

The resonant frequency of a mechanical system can be easily determined if a sufficient mathematical model is known (Qiu, 2020). The most important part in modeling the mechanical subsystem of EMS is the model of steel wire ropes. Ropes can be modeled in several ways (Vladic, 2011). In addition to modeling, resonant frequencies, i.e. vibrations, can be suppressed or detected in other ways.

There are experimental methods for detecting the resonance frequency that are rarely possible to apply to a real system (Petriková, 2012). On the other hand, there are also complex observers as a tool and method for determining the parameters of the mathematical model from which the resonant frequency is calculated (Othman, 2021). A similar method is based on a controller that dissipates vibration energy (Bao, 2011).

The elevator as a mechatronic system can be represented by the block diagram shown in Fig. 1. A modern elevator consists of two basic units: traffic control and the EMS. The part of the modern elevator that is in charge of traffic control is based on a computer controlled traffic manager that can manage one as well as several elevators in a group (Hangli, 2020).

Figure 1. The traffic control and the elevator mechatronic system as a parts of a modern elevator system



The basic part of a modern elevator is the EMS, Fig. 1. It consists of a mechanical subsystem represented by a kinematic scheme, a drive subsystem consisting of an power converter and a motor, and a control subsystem in which the control program is implemented.

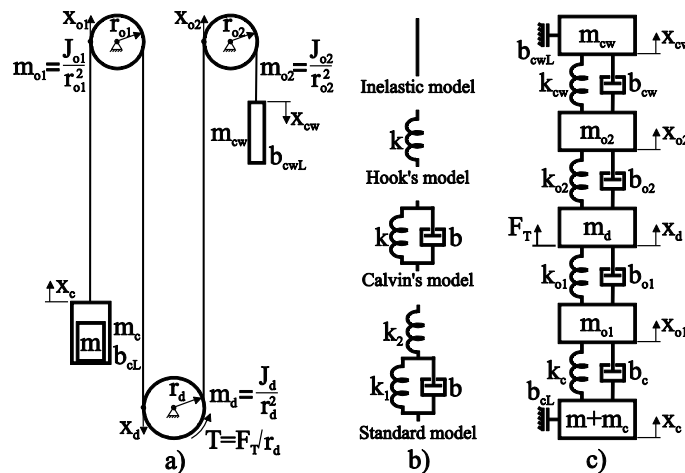
The mechanical subsystem is represented by the kinematic scheme shown in Fig. 2a. Constituent parts are wire rope, elevator car, counterweight, overhead beams and drive beam. Modern elevator drives do not have gears, so the drive beam is mounted directly on the shaft of the motor. Electric elevators are also called friction elevators because the movement of the drive beam is transmitted to the rope by the friction force. Overhead beams are used for easier energy transfer from the drive to the elevator car and to counterweight.

This paper describes the derived state space model of the EMS mechanical subsystem. Derived dynamical model is tested in computational simulations and compared with results obtained on the small-scale model of the EMS. Comparison results are proved that derived and presented mathematical model is well suited and accurate.

2. MATERIALS AND METHODS

The small-scale EMS which is used for verification of the derived state space model was in detail described in (Knezevic, 2017). In practice and theory, there is a large number of kinematic schemes that define the way of hanging the elevator cabin and counterweights. The state space derived and presented in this paper is shown in Fig. 2a.

Figure 2. a) The kinematic scheme of the EMS, b) Models of the steel wire rope, c) The equivalent mechanical scheme



The most important part in the modeling of the kinematic scheme is the modeling of the rope as an elastic element. The dynamic model of wire ropes can be expressed in several ways. The rope model as an inelastic rigid body may be satisfactory in some applications, but it is not suitable for the derivation of a dynamic model. Hook's ideal elastic body and Standard model are often used in modeling, but in the case described in this paper they are not adequate.

Hook's ideal elastic body has no modeled damping, and the Standard Model is too complex to parameterize. For derivation of the dynamic model of the mechanical subsystem of the EMS, a steel wire rope model is presented with a spring of great stiffness and with damping, Fig. 2b. In fact, it is the Calvin's model that is very often used in the literature and has proven to be satisfactory (Sandilo, 2014).

Figure 2c shows the equivalent mechanical scheme of the kinematic scheme shown in Fig. 2a. An elevator car and a counterweight are lumped masses. Overhead sheaves and a drive sheave are inertia masses. The steel wire rope is composed of great stiffness and damping while mass is neglected (Fig. 2b).

3. RESULTS

According to Newton's second law, for each mass from Fig. 2c one linear differential equation is written. Therefore, a dynamical model of the mechanical subsystem of the EMS is a 5 DOF system and it is described for all masses as follows:

$$\begin{aligned}
 m + m_c : (m + m_c)\ddot{x}_c + k_c(x_c - x_{o1}) + b_c(\dot{x}_c - \dot{x}_{o1}) + b_{cL}\dot{x}_c &= 0, \\
 m_{o1} : m_{o1}\ddot{x}_{o1} + k_{o1}(x_{o1} - x_d) + b_{o1}(\dot{x}_{o1} - \dot{x}_d) + k_c(x_{o1} - x_c) + b_c(\dot{x}_{o1} - \dot{x}_c) &= 0, \\
 m_d : m_d\ddot{x}_d + k_{o1}(x_d - x_{o1}) + b_{o1}(\dot{x}_d - \dot{x}_{o1}) + k_{o2}(x_d - x_{o2}) + b_{o2}(\dot{x}_d - \dot{x}_{o2}) &= F_T, \\
 m_{o2} : m_{o2}\ddot{x}_{o2} + k_{o2}(x_{o2} - x_d) + b_{o2}(\dot{x}_{o2} - \dot{x}_d) + k_{cw}(x_{o2} - x_{cw}) + b_{cw}(\dot{x}_{o2} - \dot{x}_{cw}) &= 0, \\
 m_{cw} : m_{cw}\ddot{x}_{cw} + k_{cw}(x_{cw} - x_{o2}) + b_{cw}(\dot{x}_{cw} - \dot{x}_{o2}) + b_{cwL}\dot{x}_{cw} &= 0,
 \end{aligned}$$

where m stands for masses, x for displacements, k stands for equivalent stiffness, b for damping coefficients and F_T stand for a driving force. The meaning of the subscripts is as follows: c indicates an elevator car, cw indicates a counterweight, $o1$ and $o2$ indicate overhead sheaves and d indicates a drive sheave. The subscripts L along b denote the friction of linear movement between the elevator car or counterweight and guide rails.

All terms in above equations as well as in Fig. 2c are linear motion terms. To obtain state space model, linear motion terms for the drive torque, the drive sheave and overhead sheaves are converted to rotational motion terms. Additionally, fitting the above equations to matrix form, state space model of the mechanical subsystem of the EMS is:

$$\begin{aligned}
 \dot{q}_m(t) &= \mathbf{A}_m q_m(t) + \mathbf{B}_m u_m(t), \\
 y_m(t) &= \mathbf{C}_m q_m(t) + \mathbf{D}_m u_m(t),
 \end{aligned}$$

where $u_m(t)$ stands for input (drive torque), $q_m(t)$ stands for a state vector and $y_m(t)$ for output (speed). An \mathbf{A}_m as a system matrix, \mathbf{B}_m as a input vector, \mathbf{C}_m as a output vector and \mathbf{D}_m as a feedthrough are given below, respectively:

$$\mathbf{A}_m = \begin{bmatrix}
 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 \frac{-k_c}{m+m_c} & \frac{-(b_c+b_{cL})}{m+m_c} & 0 & 0 & 0 & 0 & \frac{k_c r_{o1}}{m+m_c} & \frac{b_c r_{o1}}{m+m_c} & 0 & 0 \\
 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & \frac{-k_{cw}}{m_{cw}} & \frac{-(b_{cw}+b_{cwL})}{m_{cw}} & 0 & 0 & 0 & 0 & \frac{k_{cw} r_{o2}}{m_{cw}} & \frac{b_{cw} r_{o2}}{m_{cw}} \\
 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & \frac{-r_d^2(k_{o1}+k_{o2})}{J_d} & \frac{-r_d^2(b_{o1}+b_{o2})}{J_d} & \frac{k_{o1} r_d r_{o1}}{J_d} & \frac{b_{o1} r_d r_{o1}}{J_d} & \frac{k_{o2} r_d r_{o2}}{J_d} & \frac{b_{o2} r_d r_{o2}}{J_d} \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
 \frac{k_c r_{o1}}{J_{o1}} & \frac{b_c r_{o1}}{J_{o1}} & 0 & 0 & \frac{k_{o1} r_d r_{o1}}{J_{o1}} & \frac{b_{o1} r_d r_{o1}}{J_{o1}} & \frac{-r_{o1}^2(k_c+k_{o1})}{J_{o1}} & \frac{-r_{o1}^2(b_c+b_{o1})}{J_{o1}} & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
 0 & 0 & \frac{k_{cw} r_{o2}}{J_{o2}} & \frac{b_{cw} r_{o2}}{J_{o2}} & \frac{k_{o2} r_d r_{o2}}{J_{o2}} & \frac{b_{o2} r_d r_{o2}}{J_{o2}} & 0 & 0 & \frac{-r_{o2}^2(k_{cw}+k_{o2})}{J_{o2}} & \frac{-r_{o2}^2(b_{cw}+b_{o2})}{J_{o2}}
 \end{bmatrix}$$

$$\mathbf{B}_m = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{J_d} & 0 & 0 & 0 & 0 \end{bmatrix}^T, \quad \mathbf{C}_m = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0] \text{ and } \mathbf{D}_m = 0.$$

The moment of inertia of the whole mechanical subsystem denoted with J_{load} as well as the load torque on the motor shaft denoted with T_{load} both depends on the mechanical subsystem parameters and a mass in the elevator car:

$$J_{load} = J_d + J_{o1} + J_{o2} + r_d^2 \cdot (m_c + m + m_{cw}),$$

$$T_{load} = r_d \cdot (m_c + m - m_{cw}) \cdot g_n.$$

The first tests were for time response. The excitation was the step change of the drive torque, and the response was the speed of the elevator car. The experiment was performed for three different load values. The results are shown in Fig. 3. The time response shows that the speed contains damped oscillations whose frequencies are equal to the resonant frequency and do not depend on the load. The amplitude of the oscillations depends on the load, Fig. 3.

Unlike the time response, the frequency analysis provides much more useful information. Therefore, a frequency analysis was performed and the results are presented in the form of Bode plots, Fig. 4.

It can be seen from the figure that the EMS has one resonant frequency. That frequency depends only on the structure of the EMS and does not depend on the load in the EMS car.

Figure 3. The speed time response on the step torque reference for various loads in the elevator car

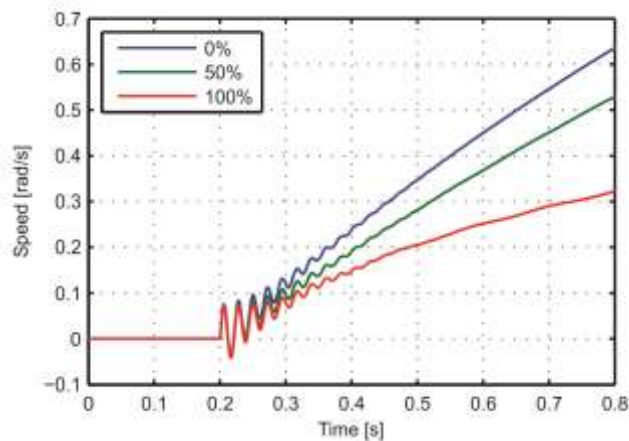


Figure 4. Bode plots from the drive torque to the speed of the EMS car for various loads

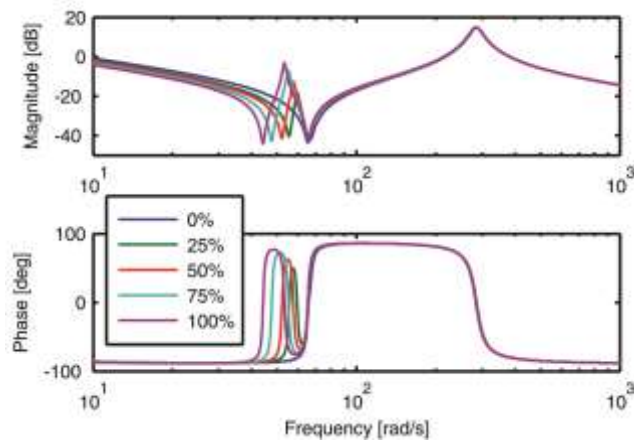
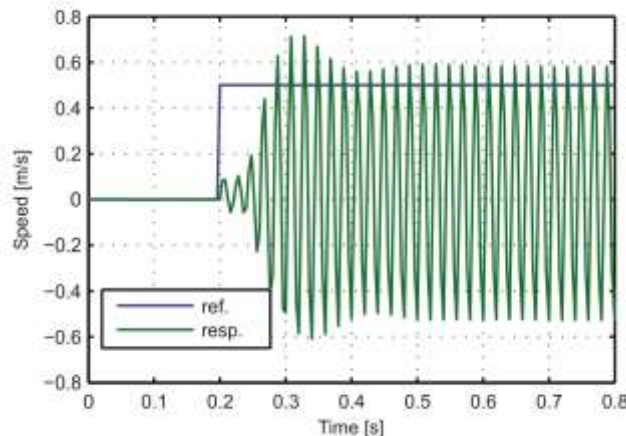


Figure 5. The time response on the step speed reference



The load in the EMS car affects the rest of the frequency response but doesn't cause any additional resonant frequencies. On the left side of the resonant frequency, the influence of the load is visible, but only on anti-resonant frequencies whose value is below zero, i.e. in the attenuation area. Frequencies out of plot are not relevant to the analysis because they are outside the bandwidth of the speed and position controller.

Figure 5. shows response on the step shaped speed reference. If the resonant frequency is inside the bandwidth of the control subsystem the speed or position control are impossible due to resonant frequencies, as it's shown in Fig. 5.

4. DISCUSSIONS

The EMS is made up of its subsystems. The mechanical subsystem is the largest and at first glance the simplest to analyze. Through research, it is already understood at the beginning that this is not the case and that the analysis of the mechanical subsystem and the phenomena that occur in it is the most complex. The mechanical subsystem consists, among other things, of flexible elements. There is the most significant influence of steel ropes, which are not rigid but elastic transmitters of mechanical movement. Due to their elasticity, they absorb and then release part of the energy that is transmitted from the shaft of the drive motor to the cabin and the counterweight. The steel ropes are predominantly, but not exclusively, the cause of vibration in EMS.

The paper presented the derived dynamic model in the state space for its easier dynamic analysis by computer simulations. A 5DOF model, i.e. five differential equations, was obtained, which, based on Cauchy's principle and by introducing variable states of the system, were adapted to be presented in matrix form.

The vibration analysis as a phenomenon in the EMS, based on dynamical model which was derived in the research, led to the conclusion that the mechanical subsystem of the elevator is characterized by its resonant frequencies. The biggest problem with resonant frequencies is that there are resonant frequencies that are within the bandwidth of the speed controller. The control signals of the speed controller are harmonically rich and contain components at frequencies equal to the resonance frequencies of the mechanical subsystem of the EMS. Therefore, the resonant circuits of the mechanical subsystem are excited by control from the control subsystem through the drive subsystem of the elevator mechatronic system. The result is vibrations that occur on the elastic ropes and are transmitted to all other components of the mechanical subsystem. Vibrations that are transmitted to the elevator cabin degrade the quality of the ride to the extent that they make it impossible to use the elevator. Apart from the visual problems, the problem with the vibrations is that they completely disable the operation of the speed controller through the speed feedback. In order to be able to determine the resonant frequency or the frequency range in which the resonant frequency is located, it is necessary to use the presented dynamic model of the EMS mechanical subsystem. It is a simple and very quick tool and a mandatory first step to synthesize the speed and position control of the EMS.

5. CONCLUSIONS

An accurate and detailed dynamic model of the EMS enables the determination of the resonance frequency of the mechanical subsystem of the EMS. The model presented in the paper is used to determine the dynamic properties of the EMS using computer simulations and to determine the existence of resonant frequencies in the bandwidth of the speed controller. The presented model has been verified and proven through experiments on a real system.

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