ESTIMATION OF FUNDAMENTAL FREQUENCY OF COMPLEX SINE SIGNAL USING q-SE AND PCC INTERPOLATION WITH 1P KEYS KERNEL ALGORITHMS - COMPARATIVE ANALYSIS

Zoran Milivojević
Academy of Technical and Educational Studies, Niš, Serbia, zoran.milivojevic@akademijanis.edu.rs
Milica Mladenović
Academy of Technical and Educational Studies, Niš, Serbia, milica.mladenovic@akademijanis.edu.rs

Abstract: In the first part of this paper, the phenomena of spectral leakage, which occurs when applying the Discrete Fourier Transformation, is described. After that, q-SE and PCC interpolation algorithm with 1P Keys kernel, which are applied to reduce spectral leakage and increase the precision of fundamental frequency estimation, are described. In the second part of the paper, the results of the Experiment, which was conducted with the aim of determining the efficiency of frequency estimation using these two algorithms, are presented. For the purposes of the Experiment, a complex sinusoidal Test signal was created. The Test signal is modified in the time domain by applying some classic, time-symmetric, window functions. Using DFT the amplitude characteristic of the Test signal was determined. By analyzing the amplitude characteristic the fundamental frequency was estimated. The results of the Experiment, estimating error MSE and the execution time of the algorithms, are presented in tabular and graphical form. Based on the experimental results, a comparative analysis was performed and the high efficiency of the PCC interpolation algorithm with the implemented 1P Keys kernel was indicated.
Keywords: frequency estimation, q-SE algorithm, PCC interpolation, interpolation kernel, 1P Keys kernel.

1. INTRODUCTION

Quick and efficient estimation of parameters of the complex sinusoidal signals, is a very important task in the field of telecommunications, radar signals, speech signals, etc., (Serbes, 2019). For this purpose, digital signal processing (DSP) is used very intensively. In many areas of the speech signal processing (speech coding, speech synthesis, speech recognition, speaker recognition, etc.) it is necessary to estimate the fundamental frequency, f_0. Estimation of the fundamental frequency is a very important and complex task. A number algorithms for estimating , f_0 have been proposed. In them, signal processing in the time and/or spectral domain is performed (Umit et al., 2006). When estimating f_0 in the spectral domain, methods based on locating peaks (peaking peaks) of the spectral characteristic and, after that, selecting the spectral component with the largest amplitude, are intensively applied. These methods are used to analyze the value of the signal in the spectrum at the frequencies at which the Discrete Fourier Transform (DFT) was calculated (Quinn, 1994). Most often, the real value of the fundamental frequency is not found at the frequencies where the DFT was calculated, but lies between two DFT samples (Pang et al., 2000).
Then, in order to equalize the energy of the signal in the time and spectral domains (Parseval's theorem), the energy is distributed to neighboring DFT spectral components. This phenomenon is called *spectral leakage*. As a result, the peak of the DFT magnitude will be shifted in relation to the real peak by a fractional part of the frequency. This causes an error in frequency estimation, which is in the interval [-(f_s / (2·N_{DFT}) Hz, (f_s /(2·N_{DFT}) Hz], where f_s is the sampling frequency and N_{DFT} is the number of points at which the DFT is calculated). One way to reduce the f_0 estimation error is to calculate the spectrum amplitude in the interval between two samples using interpolation. With this procedure, the continuous spectrum is reconstructed on the basis of discrete DFT components. Further, the analysis of the spectrum is performed by analytical procedures (differentiation, integration, extreme values,...). Today, interpolation techniques based on convolution are intensively used (Savić et al., 2022). Interpolation methods can be divided into: a) non-iterative and b) iterative methods. Non-iterative methods are mainly based on the analysis of the spectral component with maximum amplitude and its neighboring spectral components (Quinn, 1994) (Quinn, 1997) (Orguner, & Candan, 2014) (Liang et al., 2016) (Fan & Qi, 2018). Iterative methods solve the interpolation through a larger number of iterative steps (Aboutanios & Mulgrew, 2005) (Candan, 2011) (Candan, 2013).
In this paper, a comparative analysis of the accuracy of fundamental frequency estimation, using: a) the iterative q-SE algorithm (Serbes, 2019) and b) the non-iterative PCC algorithm with the implemented 1P Keys kernel (Keys, 1981), was performed. As criteria for comparative analysis: a) interpolation precision and b) execution time, were used. First, the effects of spectrum leakage are described using an Example, in which a complex signal is generated with a fundamental frequency in the range f_0 = 125 - 140,625 Hz, and a sampling frequency f_s = 8kHz. The range limits are the eighth (125 Hz) and ninth (140,625 Hz) DFT components of the spectrum. The DFT is of length N_{DFT} = 512. After that, the fundamental frequency estimation algorithms, q-SE and PCC with 1P Keys kernel, are

described. The second part of the paper describes the Experiment in which the following were tested: a) accuracy of frequency estimation and b) fundamental frequency estimation time. The Test signal is modified by classic, time-symmetric, window functions: a) Hamming, b) Hanning, c) Blackman, d) Boxcar, e) Kaiser, f) Triang, g) Gausswin, h) Bartlett, i) Bohmanwin, and j) Tukeywin. As a measure of the precision of the estimation MSE was used. The execution speed was tested on a PC (Processor Intel(R) Pentium(R) CPU G3220 @3.00 GHz, RAM 8.00 GB, OS: Windows 10 Enterprise). The speed test program was implemented in Matlab and the *tic* and *toc* commands were applied. The test results (MSE and execution time) are presented using graphs and tables. At the end, a comparative analysis of the results was performed.

The further organization of this paper is as follows. In Section 2, the q-SE and 1P Keys PCC algorithm are described. In Section 3, the Experiment is described, the results of the Experiment are presented, and a comparative analysis is performed. Section 4 is the Conclusion.

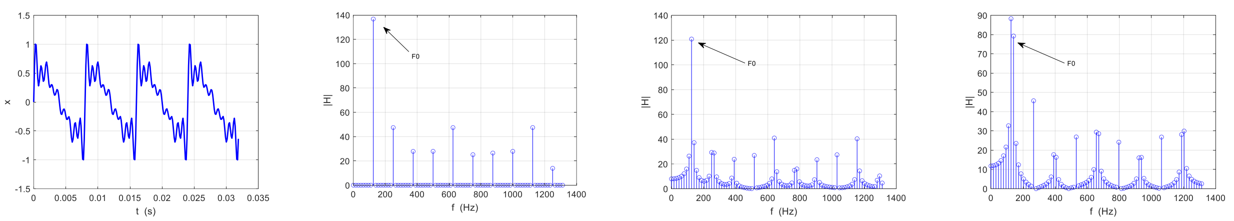
2. FUNDAMENTAL FREQUENCY ESTIMATION ALGORITHMS

2.1. Spectral leakage

Signal analysis in the spectral domain involves the application of DFT. DFT is used to calculate the amplitude and phase characteristic of the signal in the N_{DFT} of spectral bins. When the analyzed signal has a frequency that differs from the frequencies at which the DFT is calculated, and in order to equalize the energy in the time and spectral domains (Parseval's theorem), energy *leakage* occurs in the spectrum on neighboring bins (spectral leakage). As an Example, in fig. 1.a shows the Test signal x in the time domain (eq. 7, f_s = 8 kHz, with K = 10 harmonics). Over the Test signal x, DFT of length N_{DFT} = 512, was applied. The frequency resolution is $\Delta f = f_s / N_{DFT}$ = 15.625 Hz. In fig. 1.b shows the amplitude characteristic for f_0 = 125 Hz. DFT calculates the spectral component at this frequency (eighth bin), so there is no spectral leakage. Figures 1.c (f_0 = 128.9063 Hz) and 1.d (f_0 = 132.8125 Hz) show that there is spectrum leakage because the signal frequency and the DFT frequency are different.

In many DSP applications, estimation of the signal frequency is required, so the processing is done in the spectral domain. The estimation problem occurs when spectral leakage occurs. A large number of frequency estimation algorithms, based on frequency interpolation between two bins, have been developed. In the further part of the paper, the following are described: a) iterative *q*-SE (Serbes, 2019) and b) non-iterative PCC algorithm with 1P Keys convolutional kernel (Pang et al., 2000) (Keys, 1981) (Milivojević et al., 2022).

Figure 1. a) time form of the Test signal. Amplitude characteristic for: b) f_0 = 125 Hz, c) f_0 = 128.9063 Hz, and d) f_0 = 132.8125 Hz.



a) b) c) d)

2.2. *q*-SE algorithm

The algorithm, which was proposed in (Serbes, 2019), is based on the interpolation of DFT coefficients, in order to estimate the fundamental frequency of a complex sinusoidal signal. The fundamental frequency is manifested as the peak of the amplitude characteristic. Peak detection is achieved by the Peak-peaking algorithm. Sifting of the DFT coefficients is performed in the range $\pm q \in [-0.5, 0.5]$ in relation to the detected peak of the amplitude characteristic. Therefore, this algorithm called *q*-shift estimator (*q*-SE). Sifting is realized by iteration. Therefore, this method is equivalent to the interpolation of the DFT coefficients 1 / |*q*| times.

The *q*-SE algorithm is implemented in the following steps:

Input: **x** - signal, N - number of samples, duzina, f_s - sampling frequency.
Output: $\hat{f}$ - estimated frequency.
Step 1: Determining the Discrete Fourier Transform: S = DFT(**x**, N).

Step 2: Determining the spectral component with the highest amplitude, i.e. index of peak of the DFT amplitude (Peak-peaking algorithm)

$$\hat{k}_p = \arg\max_k P(k), \tag{1}$$

where $\hat{k}_p$ is the index of the peak of DFT amplitude. The amplitude of the spectral component is

$$P(k) = \frac{1}{N}\left|\sum_{n=0}^{N-1} x(n)\cdot e^{-j\frac{2\pi}{N}nk}\right|^2. \tag{2}$$

Step 3: Setting the residual frequency $\delta = 0$ (deviation of the real frequency from the DFT frequency).
Step 4: Number of iteration steps to guarantee the uniform convergence of frequencies in the range [-0.5, 0.5]:

$$Q = \begin{cases} 3, & \left\lceil \log_3\left(\log_2\left(\frac{N}{\ln N}\right)\right)\right\rceil < 3, \\ \left\lceil \log_3\left(\log_2\left(\frac{N}{\ln N}\right)\right)\right\rceil, & otherwise \end{cases} \tag{3}$$

FOR $i = 1$ to Q

Step 5: Determination of residual frequency δ:

$$\hat{\delta}_i = \frac{1}{c(q)}\cdot \mathrm{Re}\left\{\frac{S_{+q}-S_{-q}}{S_{+q}+S_{-q}}\right\} + \hat{\delta}_{i+1}, \tag{4}$$

where $S_{\pm q} = \sum_{n=0}^{N-1} x(n)\cdot e^{-j\frac{2\pi}{N}n\left(\hat{k}_p+\hat{\delta}_{i-1}\pm q\right)}$ and $c(q) = (1 - \pi\cdot q\cdot\cot(\pi\cdot q)) / (q\cdot\cos^2(\pi\cdot q))$.

END i

Step 7: The estimated frequency is

$$\hat{f} = \frac{\hat{k}_p + \hat{\delta}_Q}{f_s} N. \tag{5}$$

2.3. PCC interpolation with 1P Keys kernel

The algorithm for estimating the fundamental frequency of the signal is described in the paper (Pang et al., 2000). The algorithm is based on the signal amplitude characteristic convolution (DFT) with a convolutional kernel. A cubic, one-parameter Keys kernel (1P Keys), was used as a convolutional kernel. Therefore, this interpolation algorithm is called Parametric Cubic Convolutional (PCC) interpolation. The paper (Keys, 1981) describes a third-order parameterized kernel, which, in the literature, is called the 1P Keys kernel. The definition of 1P Keys kernel is:

$$r(f) = \begin{cases} (\alpha+2)|f|^3 - (\alpha+3)|f|^2 + 1, & |f| \le 1, \\ \alpha|f|^3 - 5\alpha|f|^2 + 8\alpha|f| - 4\alpha, & 1 < |f| \le 2 \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

where α is the kernel parameter. The PCC algorithm consists of the following steps: *Step 1*: Determination of the amplitude characteristic of the signal, $X = \mathrm{DFT}(x)$, *Step 2*: Locating the spectral bin (Pick-peaking algorithm) with the largest amplitude $X(k)$, *Step 3*: Determination of interpolation function PCC by interpolation $X_r(f) = \mathrm{PCC}(X, r) = \sum_{i=k-L}^{k+L+1} p_i \cdot r(f-i)$, where $k \le f \le (k+1)$, $p_i = X(i)$, $r(f)$ is the kernel of interpolation and L the number of samples that participate in interpolation. *Step 4*: The estimated frequency of the complex signal is $f_e = \arg\max_f (X_r)$.

3. EXPERIMENTAL RESULTS AND ANALYSIS

3.1. Experiment

In order to perform a comparative analysis of the parameters of interpolation algorithms: a) q-SE and b) PCC interpolation with 1P Keys kernel, an Experiment was carried out. In the experiment, the analysis of: a) the precision of the estimation of the fundamental frequency of the complex sine signal, and b) the speed (time) of the execution of the algorithms, was performed. Simulation Test signal, which was used to estimate the fundamental frequency f_0, was generated according to (Milivojević et al., 2010):

$$x(t) = \sum_{i=1}^{K} a_i \sin\left(2\pi\cdot i\left(f_o + g\frac{f_s}{N_{DFT}\cdot M}\right)t + \theta_i\right), \tag{7}$$

where f_0 is fundamental frequency, θ_i and a_i are phase and amplitude of the i-th harmonic, K is the number of harmonics, M is the number of points between the two samples in spectrum. In the simulation process f_0 is in the range of the k-th and $(k+1)$ DFT spectral components, θ_i are random variables with uniform distribution in the range $[0, 2\pi]$, f_s is the sampling frequency, and N is the length, that is, the number of samples of the Test signal. As a measure of the precision of the estimation of the fundamental frequency f_0, the objective MSE measure was used. In order to minimize the MSE, the kernel parameter α was optimized. MSE was calculated according to the following algorithm:

Input: N_{DFT} - number of spectral bins in which DFT is calculated, K - number of harmonics, M - number of components between two samples, a_i - amplitude of the harmonic, and θ_i phase of the harmonic, K_{DFT} - component of DFT, f_s - sampling frequency, w - window function, (α_l, α_r) - limits of the range, $\Delta\alpha$ - step.
Output: $MSE_{q\text{-}SE}$, MSE_{1P_Keys}
Step 1: Frequency resolution $\Delta f = f_s / N_{DFT}$.
FOR $\alpha = \alpha_l, : \Delta\alpha : \alpha_r$
FOR $f_0 = K_{DFT} \cdot \Delta f : \Delta f / M: (K_{DFT} + 1) \cdot \Delta f$
Step 2: Creation of Test signal $x(t)$ according to eq. 7.
Step 3: Modifikacija signala prozorskom funkcijom $x_w = x \cdot w$.
Step 4: Calculation of the spectrum using DFT in N_{DFT} of frequency points: $X = DFT\left(x_w, N_{DFT}\right)$.
Step 5: Estimation of the fundamental frequency $f_{0q\text{-}SE}$ using the q-SE algorithm (Section 2.1)
Step 6: Estimation error: $e_{f_0\_q-SE} = f_0 - f_{0q-SE}$
Step 7: Estimation of the fundamental frequency f_{0_Keys} using the PCC 1P Keys algorithm (Section 2.2).
Step 8: Estimation error: $e_{f_0\_Keys} = f_0 - f_{0\_Keys}$
END f_0
Step 8: Mean squared error of the q-SE algorithm for parameter α: $MSE_{q-SE,\alpha}(\alpha) = \overline{e_{f_0\_q-SE}}$.
Step 9: Mean squared error of the PCC 1P Keys algorithm for parameter α: $MSE_{q\_Keys,\alpha}(\alpha) = \overline{e_{f_0\_Keys}}$
END α
Step 10: Mean squared error of the q-SE algorithm $MSE_{q-SE} = \overline{MSE_{q-SE,\alpha}}$.
Step 11: Mean squared error of the PCC 1P Keys algorithm $MSE_{q\_Keys} = \min\left(MSE_{q\_Keys,\alpha}\right)$.

The parameters used in the Experiment were: f_S = 16 kHz, N_{DFT} = 512, K_{DFT} = 8, f_0 = (125 - 140.625) Hz, K = 10, M = 100, a_i = (0.98, 0.34, 0.2, 0.2, 0.34, 0.18 , 0.19, 0.2, 0.34, 0.1) V. The windows used are: a) Hamming, b) Hanning, c) Blackman, d) Boxcar, e) Kaiser, f) Triang, g) Gausswin, h) Bartlett, i) Bohmanwin, and j) Tukeywin. The described Experiment algorithm was implemented in Matlab. The speeds (time) of the fundamental frequency estimation algorithms (Step 5, Step 7), that is, the algorithm execution times (t_{Keys} , $t_{q\text{-}SE}$) were calculated using the *toc* and *tic* commands of Matlab. 5000 iterations were performed and the mean value of the execution time was determined. The Experiments were performed on a PC: Processor Intel(R) Pentium(R) CPU G3220 @3.00 GHz, RAM 8.00 GB, OS: Windows 10 Enterprise. The results are presented tabularly and graphically. Based on the results, a comparative analysis was performed.

3.2. Results

MSE values depending on the window function and alpha parameter for the Keys kernel, as well as for the q-SE algorithm, are shown in: a) fig. 2.a (Hamming), b) fig. 2.b (Hanning), c) fig. 2.c (Blackman), d) fig. 2.d (Boxcar), e) fig. 2.e (Kaiser), f) fig. 2.f (Triang), g) fig. 2.g (Gausswin), h) fig. 2.h (Bartlett), i) fig. 2.i (Bohmanwin), j) and fig. 2.j (Tukeywin). Table 1 shows the optimal values of the kernel parameter α_{opt} and minimum MSE for 1P Keys and q-SE algorithm, depending on window function. The results during the execution of the Step 5 (q-SE) and Step 7 (1P Keys) are: $t_{Keys} = 4.7102 \cdot 10^{-06}$ s, b) $t_{q\text{-}SE} = 1.108492262805 \cdot 10^{-03}$ s.

Figure 2. MSE for window functions: a) Hamming, b) Hanning, c) Blackman, d) Boxcar, e) Kaiser, f) Triang, g) Gausswin, h) Bartlett, i) Bbohmanwin, and j) Tukeywin.

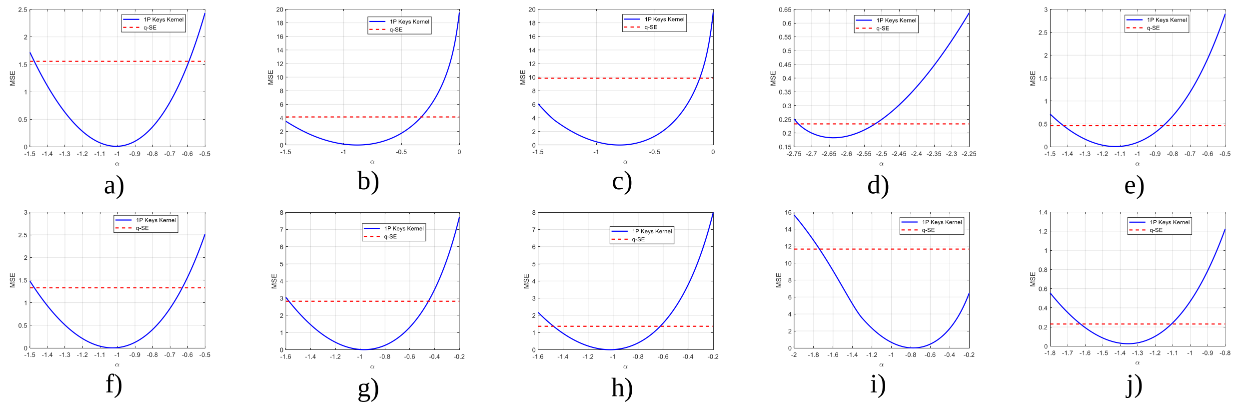


Table 1. Optimum parameter α_{opt} and minimum MSE for 1P Keys and q-SE algorithm, depending on window function.

Window	**1P Keys**		**q-SE**
	α_{opt}	**MSE_{min_Keys}**	**$MSE_{q\text{-}SE}$**
Hamming	-1.0100	0.0089	1.5550
Hanning	-0.8800	$6.4892 \cdot 10^{-04}$	4.1367
Blackman	-0.8000	$4.4867 \cdot 10^{-04}$	9.8445
Boxcar	-2.6400	0.1828	0.2333
Kaiser	-1.1300	0.0053	0.4614
Triang	-1.0300	0.0014	1.3286
Gausswin	-0.9700	0.0043	2.8212
Bartlett	-1.0200	0.0015	1.3642
Bohmanwin	-0.7800	$3.9373 \cdot 10^{-04}$	11.6464
Tukeywin	-1.3600	0.0255	0.2311
		$\overline{MSE_{1P\_Keys}}$ =0.0231	$\overline{MSE_{q-SE}}$ = 3.3622

3.3. Analysis of results

Based on the results shown in fig. 2 and Table 1, it is concluded that:

1. Compared to the estimation error of the fundamental frequency by the q-SE algorithm ($MSE_{q\text{-}SE}$), the estimation error of PCC interpolation with the implemented 1P Keys kernel (MSE_{min_Keys}), using window functions, is smaller: a) Hamming $MSE_{q\text{-}SE}$ / MSE_{min_Keys} = 1.5550 / 0.0089 = 174.71, b) Hanning 4.1367 / $6.4892 \cdot 10^{-04}$ = 6374.74, c) Blackman 9.8445 / $4.4867 \cdot 10^{-04}$ = 21941.51, d) Boxcar 0.2333 / 0.1828 = 1.27, e) Kaiser 0.4614 / 0.0053 = 87.05, f) Triang 1.3286 / 0.0014 = 949.00, g) Gausswin 2.8212 / 0.0043 = 656.09, h) Bartlett 1.3642 / 0.0015 = 909.46, i) Bohmanwin 11.6464 / $3.9373 \cdot 10^{-04}$ = 2.9579.66, and j) Tukeywin 0.2311 / 0.0255 = 9.06, times.
2. The smallest error when applying the 1P Keys kernel is with the Bohmanwin window function ($MSE_{min\_ Bohmanwin}$ = $3.9373 \cdot 10^{-04}$). The smallest error when applying *q*-SE is with Tukeywin window function ($MSE_{min\_ Tukeywin}$ = 0.2311), so the precision is higher $MSE_{min\_ Tukeywin}$ / $MSE_{min\_ Bohmanwin}$ = 586.9504 times.
3. The mean value of the errors MSE is higher with *q*-SE $\overline{MSE_{q-SE}}$ / $\overline{MSE_{1P\_Keys}}$ = 3.3622 / 0.0231 = 145.4311 times.
4. The ratio of speeds, that is execution time, is greater with *q*-SE RateTime = $t_{q\text{-}SE}$ / t_{Keys} = $1.1084 \cdot 10^{-03}$ / $4.71 \cdot 10^{-06}$ = 235.33 times.

Based on the conducted comparative analysis, it is concluded that the estimation of the fundamental frequency of the complex sine signal using PCC interpolation with the implemented 1P Keys kernel, is more accurate compared to the application of the *q*-SE algorithm, and that the algorithm is executed faster. Based on the derived conclusion, the application of the PCC interpolation algorithm with the implemented 1P Keys kernel in Real-time systems is recommended.

4. CONCLUSIONS

In this paper, the efficiency of estimating the fundamental frequency of complex sinusoidal signal, using *q*-SE and PCC algorithm with implemented 1P Keys kernel, is done. Efficiency was evaluated based on the precision of the estimation of frequency and the time execution of the algorithms. For this purpose, the Experiment was carried out. A detailed comparative analysis of the results of the Experiment show that the PCC algorithm with the implemented 1P Keys kernel has a higher precision 145.4311 times. In addition, the time execution of the PCC algorithm is 235.33 times lower. Based on the presented results and the conducted comparative analysis, the application of PCC algorithm with 1P Keys kernel in Real-time systems is recommended.

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